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THRUSTING TRAJECTORY MINIMIZATION PROGRAM FOR ORBITAL TRANSFER MANEUVERS

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16. Abstract

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The equations of motion for the transfer trajectory are those of a spacecraft maneuvering with constant thrust and mass-flow rate in the neighborhood of a single body. The thrust vector is allowed to rotate in a plane with a constant pitch rate. The transfer trajectory is characterized by six control parameters and the final orbit is determined or partially determined by the desired target parameters. The program is capable of varying from one to six control parameters to find the desired target parameters which are chosen from a large set. If the number of target parameters is less than the number of control parameters, the fuel required for the maneuver is minimized. To conserve computer time the equations of motion are integrated by a truncated power series in time. The use of the program is illustrated with three sample computer cases.

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SUMMARY

A computer program has been designed which determines the minimum-burn-time, thrusting, transfer trajectory between two Keplerian orbits. The minimization equations are formulated with constant Lagrange multipliers and solved numerically with the Newton-Raphson method. The solution obtained in this paper is not truly optimum because the control vector has been restricted to constant values (i.e., to be optimum the control vector should be a function of time).

The equations of motion for the transfer trajectory are those of a spacecraft maneuvering with constant thrust and mass-flow rate in the neighborhood of a single body. The thrust vector is allowed to rotate in a plane with a constant pitch rate. The transfer trajectory is characterized by six control parameters and the final orbit is determined or partially determined by the desired target parameters. The program is capable of varying from one to six control parameters to find the desired target parameters which are chosen from a large set. If the number of target parameters is less than the number of control parameters, the fuel required for the maneuver is minimized. To conserve computer time the equations of motion are integrated by a truncated power series in time. The use of the program is illustrated with three sample computer cases.

INTRODUCTION

In order to study completely an interplanetary, orbital mission it is necessary to define the maneuver targeting logics that are required at each of the guidance junctures. In the case of the Viking mission it is felt that adequate interplanetary targeting studies can be accomplished with the Mark IV Error Propagation Program (ref. 1) or the Simulated Trajectories Error Analysis Program (ref. 2). Both of these programs include targeting options to determine the midcourse velocity corrections such that the trajectory constraints are satisfied at the planet. In addition, each program can be used to perform an error analysis of the interplanetary phase of the mission. However, since neither program has a finite burn maneuver capability, they will not solve the targeting problem for the Mars orbit insertion maneuver, the orbit trim maneuvers, or the deorbit maneuvers. Various programs exist for the near planet targeting analysis required for this mission (e.g., refs. 3, 4, and 5); however, for various reasons the existing programs were unsuitable for this analysis. References 3, 4, and 5 are all for a coplaner transfer which is not general enough for the Viking mission. Many other existing programs are impulsive and therefore one cannot estimate fuel requirements accurately with them. For these reasons a computer program VITAP (Viking Targeting Analysis Program) was developed which solves the near Mars phase of the targeting analysis.

One of the areas of investigation in Project Viking is that of determining the minimum-fuel, thrusting, transfer trajectory between two Keplerian orbits. This problem exists for the Mars orbit insertion maneuver, the orbital trims, and for the deorbit maneuver. The equations of motion for the transfer trajectory are those of a spacecraft maneuvering with constant magnitude thrust and mass-flow rate in the neighborhood of a single body. To increase the flexibility of the program the thrust vector is allowed to pitch at a constant rate; however, for Project Viking, the pitch rate is zero. Therefore, the problem considered here is as follows: Given an initial Keplerian orbit and the characteristics of the engine, determine a set of control parameters which define the thrusting maneuver such that the required fuel is minimized and the resulting orbit satisfies a number of constraints.

A description of the mathematics used in VITAP is presented. It consists of solving a finite-dimensional minimization problem with equality constraints. The minimization equations are formulated with constant Lagrange multipliers and solved numerically with the Newton-Raphson method. Since these equations are very complicated, the first and second partial derivatives necessary for the solution are computed numerically. A discussion of this procedure is also included. A detailed description of the assumed model of the thrusting spacecraft is given together with the associated equations of motion. To conserve machine time these equations are integrated by a truncated power series in time. The power series solution to the equations of motion is completely developed in the appendix. A description of the computer program is given with a discussion of the program input and output. The use of program VITAP is illustrated with three sample computer cases.

SYMBOLS

A	submatrix defined in equation (8)
a	semimajor axis, kilometers
a^*	specific value of a, kilometers
a(t)	magnitude of thrust acceleration at time t, kilometers/second^2

B	submatrix defined in equation (8)
$\vec{B}$	vector from center of planet to incoming hyperbolic asymptote, kilometers
$\vec{B}\cdot\hat{R}$	component of $\vec{B}$ in $\hat{R}$ direction, kilometers (see sketch 3)
$\vec{B}\cdot\hat{T}$	component of $\vec{B}$ in $\hat{T}$ direction, kilometers (see sketch 3)
e	eccentricity
F	augmented function (see eq. (1))
$f(\vec{\alpha})$	function of $\vec{\alpha}$
G	lighting angle at landing point, degrees (see sketch 2)
g	partial derivatives of F with respect to $\vec{\alpha}$ (see eq. (4))
i	inclination, degrees
i^*	specific value of i, degrees
K	step size control variable (see eqs. (11))
m	mass, kilograms
N	number of trajectory integration increments
R	error variable (see eq. (12))
$\hat{R}$	unit vector perpendicular to planet equator (see sketch 3)
r	radius from center of planet, kilometers
r_a	radius of apoapsis, kilometers
r_p	radius of periapsis, kilometers
$\hat{S}$	unit vector parallel to incoming hyperbolic asymptote (see sketch 3)

$\hat{T}$	unit vector in planet equator perpendicular to $\hat{S}$ (see sketch 3)
t	time, seconds
t_b	time duration of thrusting maneuver, seconds
V_∞	hyperbolic excess velocity, kilometers/second
$\vec{X}_f$	six-dimensional state of spacecraft at end of maneuver
x,y,z	rectangular Cartesian coordinates, kilometers
α,β,δ	angles defining direction of thrust, degrees (see sketch 1)
$\vec{\alpha}$	vector of control variables
$\vec{\alpha}_1$	first guess for $\vec{\alpha}$
ΔV	integral of acceleration due to thrust, kilometers/second (see eq. (18))
$\Delta\vec{\alpha} \equiv \vec{\alpha} - \vec{\alpha}_1$	
$\Delta\alpha_1^*,\Delta\alpha_2^*$	components of vector $\Delta\vec{\alpha}^*$
$\Delta\vec{\alpha}^*$	maximum allowable step size of $\vec{\alpha}$
$\Delta\vec{\lambda} \equiv \vec{\lambda} - \vec{\lambda}_1$	
$\delta\vec{\alpha}$	infinitesimal variation of $\vec{\alpha}$
ϵ	convergence criteria (see eq. (12))
θ	angle between landing point and periapsis, degrees (see sketch 2)
$\dot{\theta}$	thrusting pitch rate, degrees/second (see sketch 1)
$\vec{\lambda}$	vector of constant Lagrange multipliers
$\vec{\lambda}_1$	first guess for $\vec{\lambda}$

μ	gravitational constant of planet, kilometers^3/second^2
ν	true anomaly, degrees
ϕ	latitude of landing point, degrees
ψ	trajectory constraints
Ω	longitude of ascending node, degrees
ω	argument of periapsis, degrees

Subscripts:

f	final value
i,j,l	indices
m	number of constraints ψ
m×m	matrix with m rows and m columns
min	minimum
n	number of control parameters
o	initial conditions
1,2,. . .	first, second, . . .

Superscripts:

T	matrix transpose
-1	matrix inverse
'	modified parameter

Dot over a symbol indicates differentiation with respect to time.

ANALYSIS

Finite-Dimensional Minimization Problem with Equality Constraints

Consider the finite-dimensional minimization problem in which equality constraints have been imposed. It is desired to find a set of controls $(\alpha_1,\alpha_2,\ldots,\alpha_n)$ such that the function $f(\alpha_1,\alpha_2,\ldots,\alpha_n)$ is minimized subject to the m constraint conditions

$$\begin{aligned}\psi_1(\vec{\alpha}) &= 0\\ \psi_2(\vec{\alpha}) &= 0\\ &\vdots\\ \psi_m(\vec{\alpha}) &= 0\end{aligned}$$

where $m \leqq n$. If f and ψ_i are sufficiently smooth and ψ_i linearly independent, the solution must satisfy the conditions

$$\frac{\partial F}{\partial \vec{\alpha}} = \begin{bmatrix}\dfrac{\partial F}{\partial \alpha_1}\\ \dfrac{\partial F}{\partial \alpha_2}\\ \vdots\\ \dfrac{\partial F}{\partial \alpha_n}\end{bmatrix} = \vec{0} \qquad \text{and} \qquad \frac{\partial F}{\partial \vec{\lambda}} = \begin{bmatrix}\dfrac{\partial F}{\partial \lambda_1}\\ \dfrac{\partial F}{\partial \lambda_2}\\ \vdots\\ \dfrac{\partial F}{\partial \lambda_m}\end{bmatrix} = \vec{0}$$

where F is defined as

$$F = f + \sum_{l=1}^{l=m} \lambda_l \psi_l \tag{1}$$

and $\vec{\lambda} \equiv \left[\lambda_1,\lambda_2,\ldots,\lambda_m\right]^T$ is an m-dimensional Lagrange multiplier vector. Therefore, necessary conditions for $\vec{\alpha}$ to minimize $f(\vec{\alpha})$ are

$$\frac{\partial F}{\partial \vec{\alpha}} = \begin{bmatrix}\dfrac{\partial f}{\partial \alpha_1}\\ \dfrac{\partial f}{\partial \alpha_2}\\ \vdots\\ \dfrac{\partial f}{\partial \alpha_n}\end{bmatrix} + \begin{bmatrix}\dfrac{\partial \psi_1}{\partial \alpha_1} & \dfrac{\partial \psi_2}{\partial \alpha_1} & \cdots & \dfrac{\partial \psi_m}{\partial \alpha_1}\\ \dfrac{\partial \psi_1}{\partial \alpha_2} & \dfrac{\partial \psi_2}{\partial \alpha_2} & \cdots & \dfrac{\partial \psi_m}{\partial \alpha_2}\\ \vdots & \vdots & & \vdots\\ \dfrac{\partial \psi_1}{\partial \alpha_n} & \dfrac{\partial \psi_2}{\partial \alpha_n} & \cdots & \dfrac{\partial \psi_m}{\partial \alpha_n}\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_m\end{bmatrix} = \vec{0} \tag{2}$$

and

$$\frac{\partial F}{\partial \vec{\lambda}} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \\ \psi_m \end{bmatrix} = \vec{0} \tag{3}$$

In addition, a sufficient condition for a local minimum (refs. 6 and 7) is given by

$$\sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \frac{\partial^2 F}{\partial \alpha_i \, \partial \alpha_j} \delta\alpha_i \delta\alpha_j > 0$$

where

$$\delta\vec{\alpha} = \vec{\alpha} - \vec{\alpha}_{min}$$

and is constrained to satisfy

$$\begin{bmatrix} \frac{\partial \psi_1}{\partial \alpha_1} & \cdots & \frac{\partial \psi_1}{\partial \alpha_n} \\ \vdots & & \vdots \\ \frac{\partial \psi_m}{\partial \alpha_1} & \cdots & \frac{\partial \psi_m}{\partial \alpha_n} \end{bmatrix} \begin{bmatrix} \delta\alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \delta\alpha_n \end{bmatrix} = \vec{0}$$

Newton-Raphson Technique

The necessary conditions for a minimum (eqs. (2) and (3)) can be solved by the Newton-Raphson technique. Let

$$g_i(\vec{\alpha},\vec{\lambda}) \equiv \frac{\partial F(\vec{\alpha},\vec{\lambda})}{\partial \alpha_i} \tag{4}$$

Then the necessary conditions can be written as

$$g_i(\vec{\alpha},\vec{\lambda}) = 0 \qquad (i = 1,2,\ldots,n) \tag{5}$$

$$\psi_l(\vec{\alpha}) = 0 \qquad (l = 1,2,\ldots,m) \tag{6}$$

Expanding equations (5) and (6) in a truncated Taylor series about the point $\vec{\alpha}_1, \vec{\lambda}_1$ (where $\vec{\alpha}_1$ denotes the first guess at the control vector and $\vec{\lambda}_1$ denotes the first guess at the Lagrange multipliers) yields

$$g_i(\vec{\alpha}, \vec{\lambda}) = g_i(\vec{\alpha}_1, \vec{\lambda}_1) + \frac{\partial g_i(\vec{\alpha}_1, \vec{\lambda}_1)}{\partial \vec{\alpha}}(\vec{\alpha} - \vec{\alpha}_1) + \frac{\partial g_i(\vec{\alpha}_1, \vec{\lambda}_1)}{\partial \vec{\lambda}}(\vec{\lambda} - \vec{\lambda}_1) = 0 \qquad (i = 1,2,\ldots,n)$$

$$\psi_l(\vec{\alpha}) = \psi_l(\vec{\alpha}_1) + \frac{\partial \psi_l(\vec{\alpha}_1)}{\partial \vec{\alpha}}(\vec{\alpha} - \vec{\alpha}_1) = 0 \qquad (l = 1,2,\ldots,m)$$

Since

$$\frac{\partial g_i(\vec{\alpha}, \vec{\lambda})}{\partial \lambda_l} = \frac{\partial}{\partial \lambda_l}\left(\frac{\partial F(\vec{\alpha}, \vec{\lambda})}{\partial \alpha_i}\right) = \frac{\partial \psi_l}{\partial \alpha_i}$$

then the necessary conditions can be expressed as

$$\left[\begin{array}{c} g_1 \\ \vdots \\ g_n \\ \hline \psi_1 \\ \vdots \\ \psi_m \end{array}\right] + \left[\begin{array}{ccc|ccc} \frac{\partial g_1}{\partial \alpha_1} & \cdots & \frac{\partial g_1}{\partial \alpha_n} & \frac{\partial \psi_1}{\partial \alpha_1} & \cdots & \frac{\partial \psi_m}{\partial \alpha_1} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial \alpha_1} & \cdots & \frac{\partial g_n}{\partial \alpha_n} & \frac{\partial \psi_1}{\partial \alpha_n} & \cdots & \frac{\partial \psi_m}{\partial \alpha_n} \\ \hline \frac{\partial \psi_1}{\partial \alpha_1} & \cdots & \frac{\partial \psi_1}{\partial \alpha_n} & & & \\ \vdots & & \vdots & & [0]_{m \times m} & \\ \frac{\partial \psi_m}{\partial \alpha_1} & \cdots & \frac{\partial \psi_m}{\partial \alpha_n} & & & \end{array}\right] \left[\begin{array}{c} \Delta\alpha_1 \\ \vdots \\ \Delta\alpha_n \\ \hline \Delta\lambda_1 \\ \vdots \\ \Delta\lambda_m \end{array}\right] = \vec{0}$$

where $\Delta\vec{\alpha} \equiv \vec{\alpha} - \vec{\alpha}_1$ and $\Delta\vec{\lambda} \equiv \vec{\lambda} - \vec{\lambda}_1$ are the corrections to improve the initial guess. The familiar Newton-Raphson iteration equations can now be written as

$$\left[\begin{array}{c} \Delta\alpha_1 \\ \vdots \\ \Delta\alpha_n \\ \hline \Delta\lambda_1 \\ \vdots \\ \Delta\lambda_m \end{array}\right] = \left[\begin{array}{ccc|ccc} \frac{\partial g_1}{\partial \alpha_1} & \cdots & \frac{\partial g_1}{\partial \alpha_n} & \frac{\partial \psi_1}{\partial \alpha_1} & \cdots & \frac{\partial \psi_m}{\partial \alpha_1} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial \alpha_1} & \cdots & \frac{\partial g_n}{\partial \alpha_n} & \frac{\partial \psi_1}{\partial \alpha_n} & \cdots & \frac{\partial \psi_m}{\partial \alpha_n} \\ \hline \frac{\partial \psi_1}{\partial \alpha_1} & \cdots & \frac{\partial \psi_1}{\partial \alpha_n} & & & \\ \vdots & & \vdots & & [0]_{m \times m} & \\ \frac{\partial \psi_m}{\partial \alpha_1} & \cdots & \frac{\partial \psi_m}{\partial \alpha_n} & & & \end{array}\right]^{-1} \left[\begin{array}{c} -g_1 \\ \vdots \\ -g_n \\ \hline -\psi_1 \\ \vdots \\ -\psi_m \end{array}\right] \tag{7}$$

Inspection of equation (7) reveals that it can be written as

$$\begin{bmatrix} \Delta\alpha_1 \\ \vdots \\ \Delta\alpha_n \\ \hdashline \Delta\lambda_1 \\ \vdots \\ \Delta\lambda_m \end{bmatrix} = \left[\begin{array}{c:c} A & B^T \\ \hdashline B & [0] \end{array}\right]^{-1} \begin{bmatrix} -g_1 \\ \vdots \\ -g_n \\ \hdashline -\psi_1 \\ \vdots \\ -\psi_m \end{bmatrix} \tag{8}$$

For the case where $m = n$, it is instructive to consider the inverse of the matrix in equation (8) using Schur's method for partitioned matrices (ref. 8), that is,

$$\begin{bmatrix} \Delta\alpha_1 \\ \vdots \\ \Delta\alpha_n \\ \hdashline \Delta\lambda_1 \\ \vdots \\ \Delta\lambda_m \end{bmatrix} = \left[\begin{array}{c:c} A^{-1} - A^{-1}B^T\left(BA^{-1}B^T\right)^{-1}BA^{-1} & A^{-1}B^T\left(BA^{-1}B^T\right)^{-1} \\ \hdashline \left(BA^{-1}B^T\right)^{-1}BA^{-1} & -\left(BA^{-1}B^T\right)^{-1} \end{array}\right] \begin{bmatrix} -g_1 \\ \vdots \\ -g_n \\ \hdashline -\psi_1 \\ \vdots \\ -\psi_m \end{bmatrix} \tag{9}$$

If $m = n$ and all the proper inverses exist, the expression $A^{-1} - A^{-1}B^T\left(BA^{-1}B^T\right)^{-1}BA^{-1}$ reduces to the null matrix $[0]$ and the corrections to the control parameters depend only on the errors in the constraints and not on the Lagrange multipliers associated with $g_i(\vec{\alpha},\vec{\lambda})$. This is to be expected when n controls are varied to satisfy n constraints and no minimization takes place. In addition, when $m = n$ the expression $A^{-1}B^T\left(BA^{-1}B^T\right)^{-1}$ reduces to B^{-1} such that

$$\begin{bmatrix} \Delta\alpha_1 \\ \vdots \\ \Delta\alpha_n \end{bmatrix} = B^{-1} \begin{bmatrix} -\psi_1 \\ \vdots \\ -\psi_m \end{bmatrix} \tag{10}$$

which are the Newton-Raphson iteration equations for solving n equations in n unknowns.

Experience has shown that the range of convergence for the Newton-Raphson technique can be considerably extended by placing constraints on the allowable size of the step to be taken. If the step size of the controls is constrained to be less than $\Delta\vec{\alpha}^*$, then a modified step size is given by

$$\left.\begin{aligned} \Delta\alpha_i' &= K^{-1}\,\Delta\alpha_i \qquad && (i = 1,2,\ldots,n) \\ \Delta\lambda_l' &= K^{-1}\,\Delta\lambda_l \qquad && (l = 1,2,\ldots,m) \end{aligned}\right\} \tag{11}$$

where

$$K = \max\left(1, \frac{|\Delta\alpha_1|}{\Delta\alpha_1^*}, \frac{|\Delta\alpha_2|}{\Delta\alpha_2^*}, \ldots, \frac{|\Delta\alpha_n|}{\Delta\alpha_n^*}\right)$$

Such a procedure will greatly increase the range of convergence for equation (9).

The Newton-Raphson technique is considered complete when the errors in the constraints and the first partial derivatives of the augmented function are small, that is, when the control vector converges and ceases to change. Symbolically, the solution is complete when

$$R < \epsilon \tag{12}$$

where

$$R = |g_1| + \ldots + |g_n| + |\psi_1| + \ldots + |\psi_m|$$

and ϵ is a small number.

Numerical Differencing Formulas

The Newton-Raphson iteration technique used to solve the finite-dimensional minimization problem requires expressions for a number of partial derivatives. From equation (7) it can be seen that the partial derivatives $\frac{\partial\psi_l}{\partial\alpha_j}$ and $\frac{\partial g_i}{\partial\alpha_j}$ are needed where

$$\frac{\partial g_i}{\partial\alpha_j} = \frac{\partial^2 f}{\partial\alpha_i\,\partial\alpha_j} + \sum_{l=1}^{l=m} \lambda_l \frac{\partial^2\psi_l}{\partial\alpha_i\,\partial\alpha_j}$$

Assuming that $f(\alpha_1,\alpha_2,\ldots,\alpha_n)$ can be differentiated directly, it remains to obtain the first and second derivatives of the constraint functions ψ_l with respect to the controls α_j. In general $\psi_l(\alpha_1,\alpha_2,\ldots,\alpha_n)$ is an implicit function of α and does not lend itself to direct differentiation. However, it was found that numerical partial derivatives are quite adequate for the Newton-Raphson iteration equations. Letting $\Delta\alpha_i$ denote a small increment in α_i, the first partial derivatives can be represented by (ref. 9, p. 136)

$$\frac{\partial\psi_l}{\partial\alpha_j} = \frac{1}{2\Delta\alpha_j}\left[\psi_l\left(\alpha_1,\ldots,\alpha_j+\Delta\alpha_j,\ldots,\alpha_n\right) - \psi_l\left(\alpha_1,\ldots,\alpha_j-\Delta\alpha_j,\ldots,\alpha_n\right)\right] \tag{13}$$

where $l = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$. Similarly, the second partial derivatives can be represented by

$$\begin{aligned}\frac{\partial^2\psi_l}{\partial\alpha_i\,\partial\alpha_j} = \frac{1}{4\Delta\alpha_i\,\Delta\alpha_j}&\Big[\psi_l\left(\alpha_1,\ldots,\alpha_i+\Delta\alpha_i,\ldots,\alpha_j+\Delta\alpha_j,\ldots,\alpha_n\right)\\ &- \psi_l\left(\alpha_1,\ldots,\alpha_i+\Delta\alpha_i,\ldots,\alpha_j-\Delta\alpha_j,\ldots,\alpha_n\right) - \psi_l\left(\alpha_1,\ldots,\alpha_i-\Delta\alpha_i,\ldots,\alpha_j+\Delta\alpha_j,\ldots,\alpha_n\right)\\ &+ \psi_l\left(\alpha_1,\ldots,\alpha_i-\Delta\alpha_i,\ldots,\alpha_j-\Delta\alpha_j,\ldots,\alpha_n\right)\Big]\end{aligned} \tag{14}$$

where $l = 1,2,\ldots,m$ and $i,j = 1,2,\ldots,n$. When j is replaced by i, equation (14) reduces to

$$\frac{\partial^2\psi_l}{\partial\alpha_i^2} = \frac{1}{\left(\Delta\alpha_i\right)^2}\left[\psi_l\left(\alpha_1,\ldots,\alpha_i+\Delta\alpha_i,\ldots,\alpha_n\right) - 2\psi_l\left(\alpha_1,\ldots,\alpha_n\right) + \psi_l\left(\alpha_1,\ldots,\alpha_i-\Delta\alpha_i,\ldots,\alpha_n\right)\right]$$

which reduces the number of times that ψ_l is evaluated. It can be shown that ψ_l must be evaluated $2mn$ times to obtain the first partial derivatives and $\left(\frac{4mn!}{2!\,(n-2)!} + 1\right)$ times to obtain the second partial derivatives.

APPLICATIONS

Equations of Motion

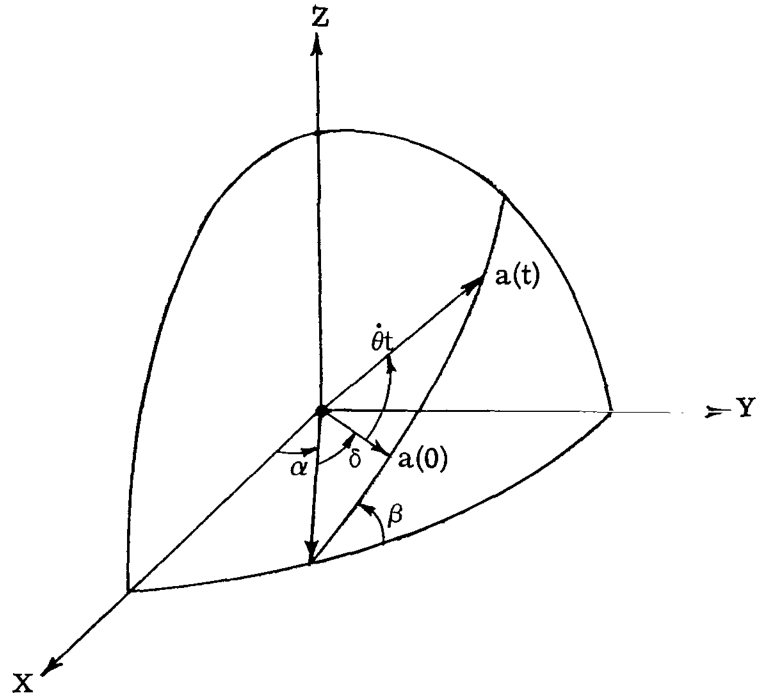
The engine used to perform these maneuvers is assumed to have a constant thrust and a constant mass-flow rate $\dot{m}$. If the mass of the spacecraft at the start of the maneuver is m_o, then the mass at time t is given by

$$m(t) = m_o + \dot{m}t \tag{15}$$

and the magnitude of the acceleration by

$$a(t) = \frac{\text{Thrust}}{m_o + \dot{m}t}$$

The direction of the acceleration vector is defined by sketch 1, where the X,Y,Z triad is the inertial Cartesian coordinate system alined such that the Z-axis is perpendicular to the Mars equator and the X-axis is at the Mars vernal equinox. This system of coordinates is normally referred to as the areocentric system. From sketch 1, it can be seen



Sketch 1

that the locus of the thrust vector is in the plane defined by the angles α and β. At the start of the maneuver (t = 0) the thrust-vector direction is defined by the angle δ and thereafter is allowed to rotate in the α,β plane at a constant rate $\dot{\theta}$ until the maneuver is terminated at $t = t_b$. During the maneuver the thrust vector is defined by the angle $\delta+\dot{\theta}t$ and the direction cosines are

$$\cos(\delta+\dot{\theta}t)\cos\alpha - \sin(\delta+\dot{\theta}t)\sin\alpha\cos\beta$$

$$\cos(\delta+\dot{\theta}t)\sin\alpha + \sin(\delta+\dot{\theta}t)\cos\alpha\cos\beta$$

$$\sin(\delta+\dot{\theta}t)\sin\beta$$

The assumed trajectory model is two body motion plus an acceleration due to thrust and is defined by the equations of motion

$$\ddot{x} = \frac{-\mu x}{r^3} + a(t) \left[\cos(\delta+\dot{\theta}t) \cos\alpha - \sin(\delta+\dot{\theta}t) \sin\alpha \cos\beta\right] \tag{16a}$$

$$\ddot{y} = \frac{-\mu y}{r^3} + a(t) \left[\cos(\delta+\dot{\theta}t) \sin\alpha + \sin(\delta+\dot{\theta}t) \cos\alpha \cos\beta\right] \tag{16b}$$

$$\ddot{z} = \frac{-\mu z}{r^3} + a(t) \left[\sin(\delta+\dot{\theta}t) \sin\beta\right] \tag{16c}$$

where

$$a(t) = \frac{\text{Thrust}}{m_o + \dot{m}t}$$

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

and μ is the gravitational constant. The initial conditions for the equations of motion are derived from the knowledge of the initial orbit plus a control variable ν_o which denotes the true anomaly on the orbit at the start of the maneuver. In other words, the initial orbit is known, but the "best" position along this trajectory to perform the maneuver is unknown. Therefore, the best position ν_o must be determined during the optimization process. In fact there are six parameters that characterize the maneuver and must be determined, namely, $\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o$. The initial direction of thrust is defined by α,β,δ; the pitch rate, by $\dot{\theta}$; the time duration of the burn by t_b; and the position of ignition, by ν_o. These six parameters constitute the set of control parameters $\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)$ for the finite-dimensional minimization problem. The function of these parameters to be minimized is simply

$$f\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right) = f\left(\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o\right) = t_b^{\,2}$$

Since the mass-flow rate is constant, minimizing the square of the burn time is equivalent to minimizing the required fuel. The function to be minimized is not restricted to $t_b^{\,2}$ and there are other continuously differentiable functions of the controls that could have been used. Once the six controls are determined they along with the initial orbit completely define the final orbit.

Trajectory Constraints

The controls are to be determined such that the burn time is minimized subject to the requirement that the final orbit has certain characteristics. Therefore, the control parameters are required to satisfy certain constraints, that is

$$\left.\begin{aligned}\psi_1\left(\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o\right) &= 0\\ \psi_2\left(\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o\right) &= 0\\ &\vdots\\ \psi_m\left(\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o\right) &= 0\end{aligned}\right\} \tag{17}$$

where $m \leqq 6$. Since the maneuver is defined by six control parameters, in general no more than six constraint functions can be satisfied. For the burn time to be minimized, the number of constraints must be less than the number of control parameters. As an example it might be required that the final orbit have a semimajor axis a equal to a constant a^* and an inclination i equal to i^*. The constraint functions would then be

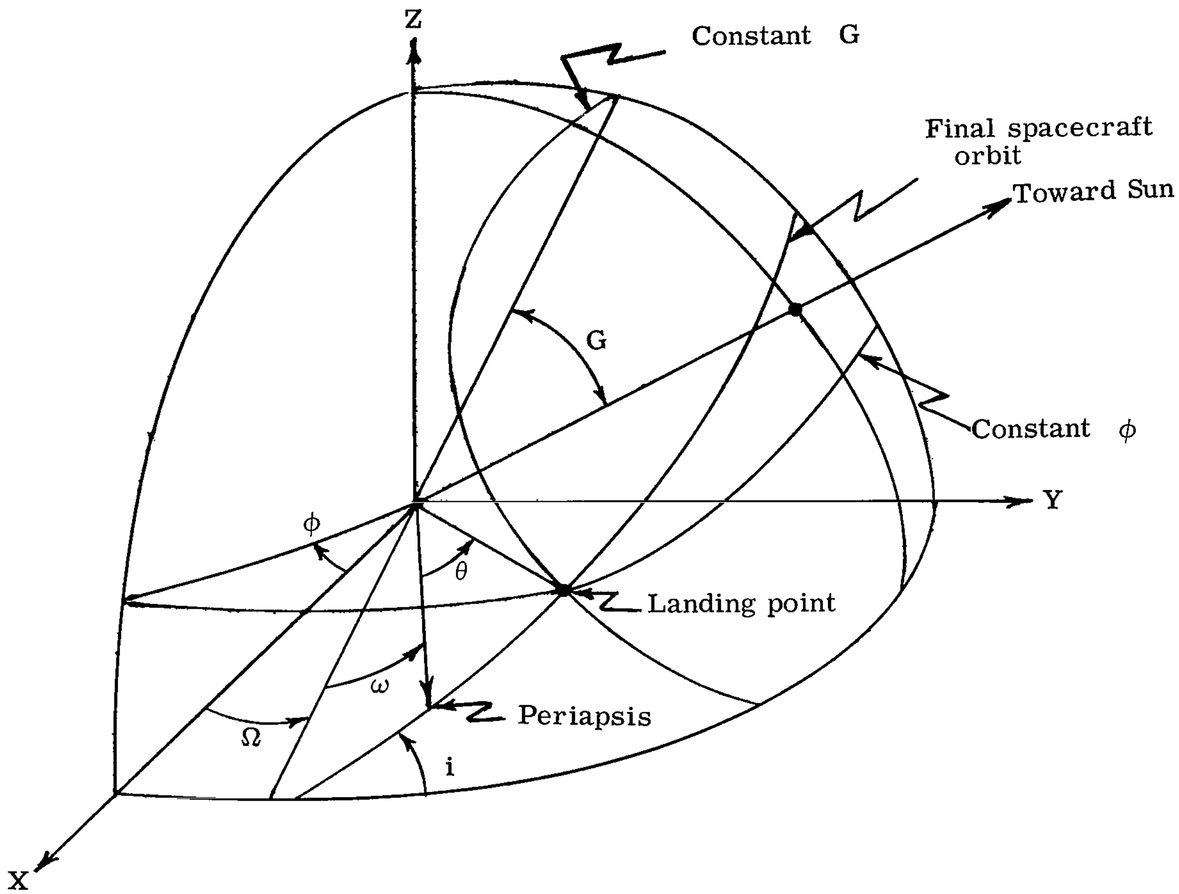
$$\psi_1 = a\left(\vec{X}_f\right) - a^* = 0$$

$$\psi_2 = i\left(\vec{X}_f\right) - i^* = 0$$

where a and i are functions of the state $\vec{X}_f$ at the end of the burn and $\vec{X}_f$ is a function of the controls through the equations of motion (eqs. (16)). Since the values of a and i are fixed in the final orbit, they are referred to as target parameters. The minimization process targets to the specified values of these parameters while minimizing the burn time.

There are any number of target parameters from which to choose. An obvious set is the six Keplerian orbital elements a, e, i, ω, Ω, and ν. Others would include the radius of periapsis r_p and the radius of apoapsis r_a. The period of the final orbit is a likely candidate. It might be required to establish the final orbit in such a manner that the spacecraft would be at a given position at some future time. Therefore, the latitude, longitude, and true anomaly of the spacecraft at a reference time should be included in this list of target parameters.

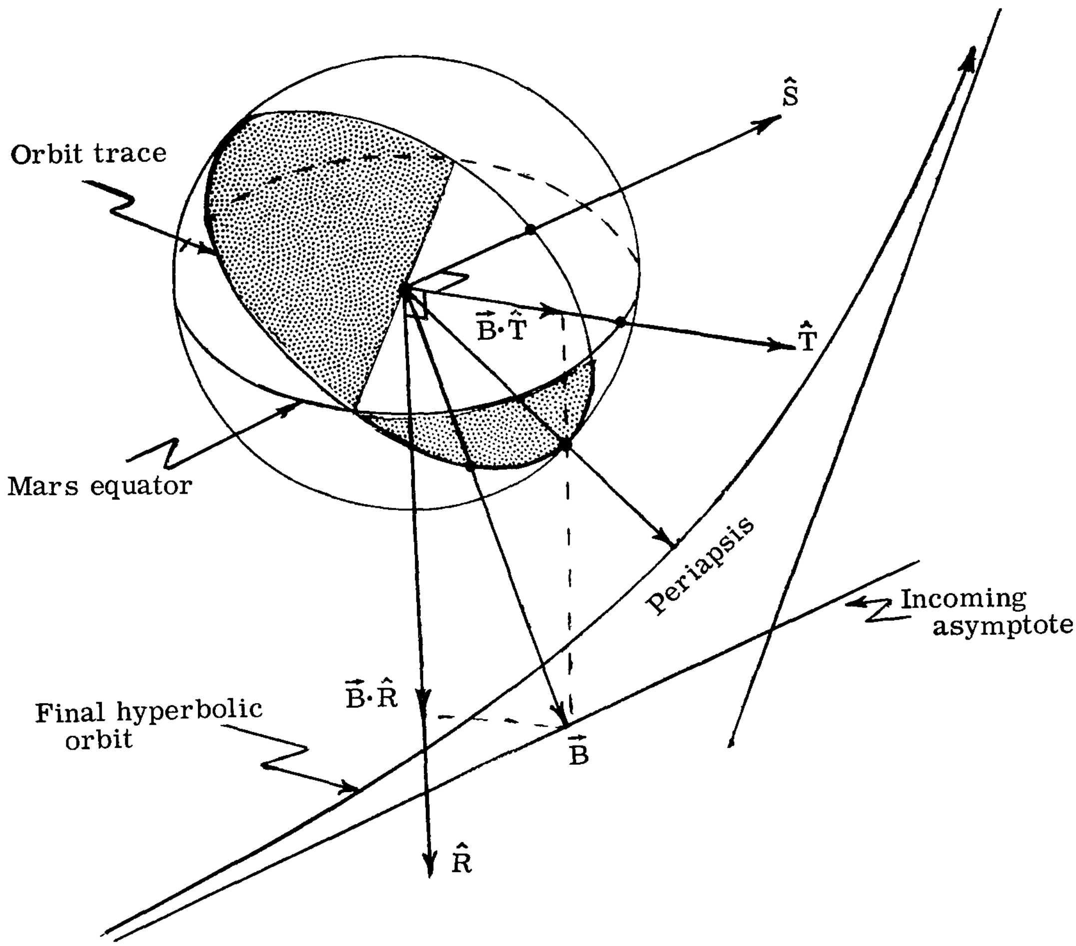
Consider the requirement that the landing point on the surface be located at a prescribed lighting condition or at a given latitude. The geometry of this situation is shown in sketch 2.



Sketch 2

The landing point is arbitrarily defined to be at an angular distance θ from periapsis and within the plane of the final orbit. Since θ is assumed constant, the landing point is a function of only the orientation angles i, ω, Ω of the final orbit. Therefore, the thrusting maneuver could be constrained to an orientation that places the landing point at a prescribed latitude ϕ. Similarly, the lighting angle G at the landing point could serve as a target parameter.

Other useful target parameters are the hyperbolic impact plane parameters $\vec{B}\cdot\hat{T}$, $\vec{B}\cdot\hat{R}$, V_∞, $\hat{S}$. These parameters are defined in sketch 3 and reference 10. The three unit vectors $\hat{S},\hat{T},\hat{R}$ form a coordinate system where $\hat{S}$ is parallel to the incoming asymptote of the spacecraft orbit, $\hat{T}$ lies in the Mars equatorial plane perpendicular to $\hat{S}$, and $\hat{R}$ completes the triad. The miss vector $\vec{B}$ is in the $\hat{R}$-$\hat{T}$ plane and represents the distance from the center of Mars to the incoming asymptote. It is



Sketch 3

usually characterized by its two components $\vec{B}\cdot\hat{T}$ and $\vec{B}\cdot\hat{R}$. This set of parameters $\vec{B}\cdot\hat{T}$, $\vec{B}\cdot\hat{R}$, $\hat{S}$ and the hyperbolic excess velocity V_∞ completely specify the final hyperbolic orbit. They are frequently more useful than the standard orbital elements and, therefore, should be included in the list of target parameters.

The choice of specific target parameters determines the constraint equations which enter the optimization process. As stated previously, no more than six constraints and at least one constraint must be imposed. The choice of the individual target parameters is important in that the ones mentioned are not all independent. As an example, suppose that the three target parameters a, e, r_p were chosen. Since a and e dictate the radius of periapsis of the final orbit, it would be impossible to satisfy these three constraints unless the third constraint happened to be consistent with that dictated by the first two. In general these three target parameters are dependent and would not constitute an acceptable set of constraints. Therefore, the specific target parameters should be chosen with care.

The areas in Project Viking which require optimum orbital transfer can now be analyzed. Given the orbital elements of the original orbit and an initial guess on the controls $\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o$ and the multipliers λ_l, the necessary equations for a minimum maneuver (eqs. (2) and (3)) subject to the desired constraints can be solved by the Newton-Raphson iteration technique (eq. (7)).

COMPUTER PROGRAM VITAP

Program Description

The equations for an optimum, thrusting transfer between two Keplerian orbits and their associated Newton-Raphson solution have been incorporated into a computer program, VITAP. It consists of a main program and 11 subroutines which are written entirely in FORTRAN computer language for the Control Data 6600 computer system. The program resulted in a field length of $50\,000_8$.

Various options have been included in VITAP which allow for a considerable amount of flexibility. The option is available to vary the six control parameters $\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o$ or to fix one or more of the controls at a constant value. For example, if the pitch rate $\dot{\theta}$ is fixed at a constant value, the transfer maneuver is optimized with respect to the remaining five control parameters. This option allows various guidance laws to be considered. In addition a number of different target parameters are available as constraints on the optimum maneuver. In fact, as many as six constraints out of a set of twenty can be selected. Thus, quite a large number of combinations of control variables and target parameters are available. Program VITAP also operates in three modes. The first mode is the normal optimum transfer outlined previously. The second mode allows for the optimum transfer with the additional constraint that the inclination of the final orbit be between an upper and lower bound. The third mode targets backwards. This mode considers the problem of finding the best orbit from which to establish a given orbit. Here the final orbit is completely known and the initial orbit is unknown, thus, the term "backward" targeting.

The computer time required to find the optimum transfer is important from a practical consideration. It is easy to imagine the large number of computations which are performed since the solution is iterative and contains the first and second numerical partials of the target parameters with respect to the controls. These numerical partials require that the equations of motion (eqs. (16)) be integrated repeatedly. If all six of the controls are free, then 12 trajectories are required to compute the first partials and 61 trajectories for the second partials. Thus, the equations of motion must be integrated 73 times for a single iteration. In general the required number of trajectories is

$\left(2n + \frac{4n!}{2!(n - 2)!} + 1\right)$ where n is the number of free control parameters. The number of iterations necessary to converge the initial guess to the optimum control vector varies considerably depending on how good the initial guess is and the sensitivity of the solution. However, when the convergence is slow, a large number of trajectories must be integrated. Although many other calculations are performed, the majority of the machine time is spent integrating these trajectories. For this reason the equations of motion were expanded in a power series in time and are integrated in this manner. (See the appendix.) This approach is much faster than the more standard numerical methods of integration. To further decrease the machine time, the trajectories are divided into N increments each of which is integrated with a 13-term power series. Thus, the duration of each segment is t_b/N where $N = 1,2,\ldots,10$. As a result of integrating by power series, the computer program is not limited by machine time.

Program Input

All input to program VITAP is accomplished by means of a FORTRAN namelist DAT. Each of the name list variables is defined in table I.

The combination of free control variables and desired target parameters is controlled by an array of 12 integers. NOPT(1) to NOPT(6) correspond to α, β, δ, $\dot{\theta}$, t_b, and ν_0. If a 1 is input in NOPT(K), then the Kth control is free to vary. A 0 denotes that the Kth control is fixed and not allowed to vary from the initial guess which is input by the GS array. For example, consider the case where the thrust vector is not allowed to pitch but remains in a constant inertial direction throughout the burn. Obviously, the three angles α,β,δ (sketch 1) overdefine the problem since only two angles are needed to define the inertial direction. To overcome this problem the angle β could be held constant at 90° allowing α and δ to function as the right ascension and declination of the thrust vector, respectively. Thus, a constant inertial maneuver requires that $\dot{\theta} = 0$ and $\beta = 90^\circ$ throughout the maneuver. The appropriate input for the first six integers of NOPT would be NOPT = 1, 0, 1, 0, 1, 1, and GS = α_1, 90., δ_1, 0., t_{b_1}, ν_{o_1} where the subscript denotes the first guess at the controls. Mathematically, the number of control variables is reduced from six to four so that the n of equation (7) is equal to four. This is the equivalent of omitting two rows and two columns of the general Newton-Raphson matrix of partial derivatives. An interesting point is the number of trajectory integrations needed for each iteration. Since only 33 trajectories are needed to calculate the partial derivatives instead of the usual 73, this case should use less than half the machine time needed for the general case. The constraints imposed on the optimum solution are defined by NOPT(7) to NOPT(12). At most VITAP will consider six constraints which would correspond to 1 for NOPT(7) to NOPT(12). However, if

NOPT(6 + K) = 0, then the Kth constraint is not invoked. In other words NOPT turns the constraints off and on by an input of 0 or 1, respectively. In this manner from one to n constraints are considered where n is the number of free controls. In this example where β and $\dot{\theta}$ were fixed the number of constraints must be four or less. The specific target parameters are denoted by KOPT, an array of six integers each of which have a value from 1 to 5. Table II defines the various options. The values of the target parameters chosen are then fixed at the values in array AIN. Therefore, the constraint is turned on by NOPT, the target parameter is defined by KOPT, and its value is input by AIN. For example, suppose

NOPT = 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0

KOPT = 1, 2, 1, 1, 1, 1

AIN = 20488., 0., 10., 0., 0., 0.

From NOPT it can be seen that $\alpha, \delta, t_b, \nu_0$ are the free controls while β and $\dot{\theta}$ are fixed. In addition two constraints are considered. KOPT shows that the two target parameters are a and i while AIN states that $a^* = 20488$ km and $i^* = 10^o$. The zeros in AIN have no function since these constraints are turned off by NOPT. However, the corresponding input in KOPT is meaningful since the values of the six target parameters chosen from table II are printed as output. Note that one of the target parameters in table II is $1/a$. This parameter is equivalent to a and provides a smooth transition between hyperbolic and elliptical orbits.

The remaining input parameters of table I need little explanation. The five initial orbital elements are defined by the CONI array. The sixth element, true anomaly, is not input since it is a control variable. The GL array is similar to GS and contains the initial guesses on the Lagrange multipliers. As mentioned previously, the rate of convergence is highly dependent on the initial guesses at the controls. However, the guesses at the multipliers seem to have little effect on the convergence. If GL is not input, VITAP fills the array with 1's which are acceptable initial values for the Lagrange multipliers. The small increments in the control parameters $\Delta\alpha_i$ used to generate the numerical partial derivatives (eqs. (13) and (14)) are defined by the HP array. The maximum allowable step size for the controls during the Newton-Raphson iteration $\Delta\alpha_i^*$ are defined by the V1 array. The step size is then computed according to equation (11). Values for both HP and V1 are built into the program and seldom need changing. However, these values may be altered at any time by the appropriate input. The number of segments used to integrate the equations of motion by power series is input through NSTEPS. If the total burn time t_b is 1800 seconds and NSTEPS is 6, then VITAP integrates in 300-seconds segments with a 13-term power series, the initial conditions for the second segment being the end conditions of the first segment. A reasonable input value for NSTEPS is one which results in about 300- to 400-second segments. The accuracy of the trajectory, however, can be improved by increasing NSTEPS at the expense of machine time. The three parameters describing the spacecraft are mass, mass-flow rate, and thrust which are input according to table I. The gravitational constant of Mars μ is stored in VITAP as UMARS = 42828.4 km^3/sec^2 but can easily be changed by input. Some of the target parameters of table II require additional input other than just the value input through the AIN array. For example, all the target parameters related to a reference date require two Julian dates, the time at which the spacecraft would reach periapsis on the initial conic and the time at which the target parameter is to have the given value. These two time parameters are PERJD and REFJD, respectively, and need be input only if the time-related target parameters are exercised. To constrain the latitude of the landing point, the angle θ must be input through PER. In addition, if the lighting angle G is one of the objects of the targeting, then the right ascension and declination of the subsolar point, SLONG and SLAT, must also be input. Once all the input is defined, the program VITAP uses the initial guesses at the controls and multipliers to start the Newton-Raphson iteration technique which continues until one of two situations occurs. If the sum of the errors (eq. (12)) is less than the value of ϵ which is input through ERR, then the process is assumed to have converged at the optimal set of controls and the solution is completed. On the other hand, if the number of iterations exceeds the input value of MAXIT, the process is considered nonconvergent and is stopped.

Finally, the mode of operation must be defined as 1, 2, or 3 by the input parameter MODE. The normal targeting mode (MODE = 1) is straight forward. The initial orbit is specified by $a_0,e_0,i_0,\omega_0,\Omega_0$ along with the desired target parameters. The program VITAP finds a control vector $\vec{\alpha}$ which maneuvers the spacecraft from the initial orbit to the final orbit which satisfies the target parameters. If there is at least one more free control than target parameters, VITAP minimizes the burn time – or equivalently the required fuel – by the Newton-Raphson technique (eq. (7)). If the number of target parameters is equal to the number of free controls, the solution is a straight forward search without minimization (eq. (10)). To target the inclination within bounds, the second mode (MODE = 2) is used. This search is similar to mode 1 with the restrictions that the inclination of the final orbit must not be specified as a target variable and the number of free controls must be greater than the number of target parameters. If the inclination constraint option NOPT(9) is not equal to 0, it will be set equal to 0. First the program finds the optimum controls which will result in some inclination i_f. This inclination is then compared to the two boundaries specified by the input parameters BOUND(1) and BOUND(2). If the inclination is between these bounds, the solution is complete; if it is outside this interval, the program retargets to the closest bound. With the addition of another target parameter i, the second solution may no longer be a minimization.

The third option (MODE = 3) is more involved than the other two. It has been termed backward targeting and finds the best orbit from which to establish a given orbit. The only change in input involves the CONI array. Instead of CONI containing the initial orbit, for mode 3 it contains the final orbit while the initial orbit is partially defined by the target parameter options of table II. Thus, the final orbit is completely defined except for the true anomaly and the initial orbit is only partially defined by the target parameter constraints. The targeting problem, then, is to find the remaining initial orbital elements such that a transfer from this orbit to the given orbit is optimum. It is solved with the same mathematics as mode 1 with some changes in procedure. The program internally changes the input and considers the motion from the final orbit to the initial orbit. When the motion is reversed the final orbital elements become a_o, e_o, $180^o - i_o$, $\Omega_o + 180^o$, and $180^o - \omega_o$ which is due to the interchange of the ascending and descending nodes. The initial mass of the spacecraft on the initial orbit is known from input, but the mass on the final orbit is not known since the optimum burn time has not been established. It is obvious that the spacecraft will lose mass during the transfer from the initial to the final orbit. Equation (15) can be used to predict the mass in the final orbit as $m_f = m_o + \dot{m} t_b$, where m_o is the initial mass and t_b is a guess at the burn time. The initial guesses at the controls are changed internally to α, β, $\delta + \dot{\theta} t_b$, $-\dot{\theta}$, t_b, $-\nu_o$ due to the reversal in motion. The program then uses the corrected orbital elements and controls to integrate the equations of motion which results in some initial orbit which neither satisfies the constraints nor is optimum. The guesses at the controls are then corrected according to the Newton-Raphson matrix, and the iteration procedure continued until the solution is obtained. The only difference between this procedure and the normal procedure is that the mass m_f in the final orbit is corrected for each iteration according to the change in burn time t_b. Therefore, the solution is complete when both the controls and mass have converged. For this reason the backward targeting converges much slower than the normal targeting.

Program Output

The computer outputs for each of the three modes of operation are presented in tables III, IV, and V.

The normal targeting mode (MODE = 1) is presented in table III. The first output is a complete listing of all of the parameters input through the namelist DAT. These parameters have been previously defined in the section entitled "Program Input" and in table I. Next is a formal listing of the mode of operation, the initial orbital elements, and a description of the free control parameters and the selected target parameters. It can be seen that the initial conic is a hyperbola and that the initial guess at the true anomaly is -60^o. The program will find a subset of four controls $(\alpha, \delta, t_b, \nu_o)$ such that the final orbit has a prescribed value of a (or $1/a$), i, and ω. Since there are more control variables than target parameters, the program will find the controls which minimize t_b^2. When four controls are varied to find three parameters in the final orbit, the operation is said to be a 4 by 3 search.

The search or iteration is recorded by a selected set of output parameters. The first, second, and last iterations are shown in table III. The control parameters for the first iteration which are the initial guesses are shown followed by the initial guesses at the Lagrange multipliers. Next, is presented the orbit which resulted from applying the initial controls to the initial orbit. The corresponding target parameters are then output which are naturally in error since the initial guess at the control variables was not the optimum set. The sensed velocity corresponding to t_b is output followed by the error in the target parameters. The sensed velocity is the integral of the acceleration due to thrust, that is

$$\Delta V = \int_0^{t_b} \frac{\text{Thrust}}{m_o + \dot{m}t} dt = \frac{\text{Thrust}}{\dot{m}} \ln\left(\frac{m_o + \dot{m}t_b}{m_o}\right) \tag{18}$$

and the errors in the target parameters are the differences between the desired values and the values obtained from the present set of controls. For example, from the input it can be seen that the inclination of the final orbit is 35^o. This is denoted by the third entry in the AIN array. However, the initial controls produced an orbit with an inclination of 33.328^o. Thus, the third target parameter is in error by 1.672^o. Note that three of the target parameters e,Ω,ν_f have zero error because the second, fifth, and sixth target options were not exercised as can be seen from the NOPT array. The errors in the target parameters are then used to improve the initial guess at the controls (eq. (7)). The corrections to the initial controls are in the next line of printout. Two points should be made about these corrections. First, no correction is added to β and $\dot{\theta}$ because zeros were input in the NOPT array in the locations corresponding to β and $\dot{\theta}$; thus, the program does not allow β and $\dot{\theta}$ to vary but keeps them fixed at the initial values which for this iteration were 90^o and 0. deg/sec, respectively. The second point is that the corrections to the controls were limited by the maximum allowable step size in ν_o. According to the V1 array, the step in true anomaly for one iteration cannot be greater than 15^o. Since the correction is exactly 15^o, this implies that the Newton-Raphson technique (eq. (7)) calculated a larger correction and that the corrections were reduced according to equation (11). At the end of this block of output is a row labeled "Eigenvalues of second partials of augmented function." This row represents the eigenvalues of the upper left-hand portion of the matrix composed of partial derivatives (eq. (7)). In other words, they are the eigenvalues of the matrix $\frac{\partial g_i}{\partial \alpha_j}$ or $\frac{\partial^2 F}{\partial \alpha_i \, \partial \alpha_j}$. These values are used as a diagnostic tool.

The printout of the second iteration follows the same format as the first iteration. The control parameters and the Lagrange multipliers have been updated by the corrections computed in the first iteration and should produce a final orbit that is closer to the optimum solution. However, due to the interplay between satisfying both the constraint equations and the necessary equations for a minimum, it is not always apparent that the succeeding iterations are closer than the preceding ones. For the case being considered the second iteration does, however, seem to be closer to the solution. The errors in two of the target parameters i, ω were reduced while the error in $1/a$ increased slightly. In addition, the sensed velocity was reduced. As before the errors in the target parameters are related to the corrections in the control variables and the iterative process continues.

The Newton-Raphson technique is repeatedly applied until both the constraint equations and the necessary equations for a minimum are satisfied (eq. (12)). The 29th iteration shows that the process has converged since the errors in the target parameters are very small and the computed corrections to the control variables are negligible. The optimum set of controls, then, are listed in the first line of printout. They not only produce a final orbit with the desired target parameters $1/a$, i, ω but also establish it with a minimum of fuel. Since the eigenvalues of the second partials of augmented function are all greater than zero, this is indeed a minimum solution. As a point of interest the Newton-Raphson matrix and its inverse (eq. (7)) are output for the last iteration. The matrices presented are the full 12 by 12 matrices. The zero rows and columns correspond to fixed control parameters and to the constraints that were not exercised. Accordingly, these matrices are reduced to the proper dimensions during the actual computations. The final output is the machine time in seconds required to compute the optimum set of control parameters.

Table IV presents a sample case of targeting such that the inclination is within bounds (MODE = 2). The second mode of operation is similar to the normal targeting mode and allows for the optimum transfer with the additional constraint that the inclination of the final orbit be between an upper and lower bound. The procedure is first to disregard the inclination constraint and solve for a set of optimum controls. If these controls produce a final orbit that satisfies the inclination inequality, then the solution is complete. If this is not the case, the program then determines which inclination bound is closest to the present solution and retargets to that value of inclination. This procedure is demonstrated in table IV. The transfer is from a hyperbola to an ellipse and is a 6 by 4 search. It is necessary that the number of control parameters exceeds the number of constraints since the inclination will be added as a constraint if it is not within bounds. Also note that inclination is a free variable for the first part of the solution and that the final orbit is specified by a, e, ω, and Ω. The program proceeds to tanget to a, e, ω, and Ω. After 12 iterations, the optimum controls are found which produce a final orbit with an inclination of 35.28°. From the input it is seen that the constraint on the inclination is $31^{\circ} \leqq i_f \leqq 35^{\circ}$. Thus, the inclination constraint was not satisfied by targeting to a, e, ω, and Ω. The program then proceeds to turn on the third constraint option (NOPT(9) = 1), define the value of the inclination constraint as the closest bound (AIN(3) = 35.), and set the initial guesses at the controls and Lagrange multipliers equal to the values obtained in the first search. These changes can be seen by comparing the first and second listing of the namelist parameters. The program then proceeds to solve this 6 by 5 search and after 6 iterations converges to the optimum set of control parameters.

The third mode of operation, backward targeting, is presented in table V. This mode (MODE = 3) considers the problem of finding the best orbit from which to establish a given final orbit. For the particular case shown the final orbit is an ellipse specified by the five orbital elements a, e, i, ω, and Ω. The initial orbit is only partially specified by three hyperbolic parameters: the hyperbolic excess velocity, the declination of the approach asymptote, and the right ascension of the approach asymptote. These three parameters are designated as target parameters. The problem, then, is to determine the remaining hyperbolic elements and a set of controls so that when these controls are applied to the approach hyperbola the resulting orbit will be the given ellipse. In addition the fuel required for the transfer is to be a minimum. The procedure, as outlined previously, is to target backwards from the final ellipse to the initial hyperbola. This reversal of motion is handled internally by the program with one exception. The initial guess on the true anomaly at the start of the burn ν_o is replaced by a guess on the true anomaly at the end of the burn. That is, the input value of ν_o is 45° which is a guess at the true anomaly in the ellipse where the burn will terminate.

The printout of the first iteration shows the result of reversing the motion and applying the intital controls to the ellipse. As would be expected the hyperbolic excess velocity, the declination of the asymptote, and the right ascension of the asymptote are all in error. These errors are used to correct the control variables. One main difference between this mode and the others is the problem associated with the mass of the spacecraft. In order to integrate the equations of motion from the ellipse to the hyperbola, the mass of the spacecraft in the ellipse must be known. The mass, however, is a function of the burn time t_b required to establish the ellipse, and this time is not known until the solution is complete. Thus, an estimate of the mass in the ellipse is computed as a function of the present value of the burn time. This estimate is included in the output under the heading "FINAL MASS." Therefore, the iteration process must converge the final mass as well as the set of control variables. This additional requirement slows the iteration process considerably. For the case considered here the program required 46 iterations to converge the initial controls to the final set of optimum controls. The "best" hyperbola for such a maneuver is given by the third line of printout.

CONCLUDING REMARKS

This report has described a program for determining the minimum-burn-time, thrusting, transfer trajectory between two Keplerian orbits. Basically, the method described involves the solution of a constrained minimization problem by use of constant Lagrange multipliers and the Newton-Raphson iteration technique. The model of the transfer trajectory allows for the thrust vector to rotate in a plane with a constant pitch rate. In all, six control parameters characterize the transfer trajectory; thus, as many as six constraints can be imposed on the final orbit. If the number of constraints is less than six, the program solves for the set of controls which minimizes the burn time or equivalently the required fuel. The option is available to fix any of the six control variables and allows various guidance laws to be investigated such as a constant inertial burn. This option together with the twenty different constraints from which to choose allows a considerable amount of flexibility. In addition the program VITAP operates in three modes. The first mode is the normal optimum transfer, the second mode is similar to the first with the additional constraint that the inclination of the final orbit be between an upper and lower bound, and the third mode targets backwards. This mode considers the problem of finding the best hyperbola from which to establish a given ellipse. These three modes of operation have been exercised extensively for representive 1975 Mars orbital insertion maneuvers.

The program uses only necessary conditions for a minimum. Accordingly, there is no guarantee that the solution obtained will be a minimum. If, however, the eigenvalues of the matrix corresponding to the second partials of the augmented function are all positive, then the solution is indeed a local minimum. This particular problem of using only necessary conditions offers no great difficulty when there is a judicious choice of trajectory constraints and reasonable initial guesses on the control parameters.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 5, 1971.

APPENDIX

POWER SERIES EXPANSION OF EQUATIONS OF MOTION

The most time consuming computation in the Viking Targeting Analysis Program is the numerical integration of the equations of motion. Since numerical partial derivatives are used in the Newton-Raphson technique, the equations of motion are integrated repeatedly. For this reason the equations of motion are numerically integrated by power series which are considerably faster than the more standard numerical methods.

The power series solution of the two body equations of motion have been solved by Schanzle (ref. 11). The governing differential equations for two body motion are

$$\ddot{x} = \frac{-\mu x}{r^3} \tag{A1a}$$

$$\ddot{y} = \frac{-\mu y}{r^3} \tag{A1b}$$

$$\ddot{z} = \frac{-\mu z}{r^3} \tag{A1c}$$

where

$$r = \left(x^2 + y^2 + z^2\right)^{1/2} \tag{A2}$$

and μ is the gravitational constant. The trajectory model considered for the Mars maneuvers requires the addition of acceleration terms which are due to the low thrust (nonimpulsive) engine. Therefore, the equations of motion for the finite burn are (eqs. (16))

$$\ddot{x} = \frac{-\mu x}{r^3} + a(t)\left[\cos\,(\delta+\dot{\theta}t)\cos\alpha - \sin\,(\delta+\dot{\theta}t)\sin\alpha\cos\beta\right] \tag{A3a}$$

$$\ddot{y} = \frac{-\mu y}{r^3} + a(t)\left[\cos\,(\delta+\dot{\theta}t)\sin\alpha + \sin\,(\delta+\dot{\theta}t)\cos\alpha\sin\beta\right] \tag{A3b}$$

$$\ddot{z} = \frac{-\mu z}{r^3} + a(t)\left[\sin\,(\delta+\dot{\theta}t)\sin\beta\right] \tag{A3c}$$

where $a(t)$ is the magnitude of the acceleration and is given by

$$a(t) = \frac{\text{Thrust}}{m_o + \dot{m}t} \tag{A4}$$

The following definitions will simplify the solution of the equations of motion:

$$b \equiv \frac{1}{r^3} \tag{A5}$$

$$A_x(t) \equiv a(t)\left[\cos(\delta+\dot{\theta}t)\cos\alpha - \sin(\delta+\dot{\theta}t)\sin\alpha\cos\beta\right] \tag{A6a}$$

$$A_y(t) \equiv a(t)\left[\cos(\delta+\dot{\theta}t)\sin\alpha + \sin(\delta+\dot{\theta}t)\cos\alpha\cos\beta\right] \tag{A6b}$$

$$A_z(t) \equiv a(t)\left[\sin(\delta+\dot{\theta}t)\sin\beta\right] \tag{A6c}$$

Therefore, equations (A3) can be written as

$$\ddot{x} = -\mu bx + A_x \tag{A7a}$$

$$\ddot{y} = -\mu by + A_y \tag{A7b}$$

$$\ddot{z} = -\mu bz + A_z \tag{A7c}$$

for which the assumed solution is

$$x = \sum_{i=0}^{i=\infty} x_i t^i \tag{A8a}$$

$$y = \sum_{i=0}^{i=\infty} y_i t^i \tag{A8b}$$

$$z = \sum_{i=0}^{i=\infty} z_i t^i \tag{A8c}$$

$$r = \sum_{i=0}^{i=\infty} r_i t^i \tag{A8d}$$

$$b = \sum_{i=0}^{i=\infty} b_i t^i \tag{A8e}$$

$$A_x = \sum_{i=0}^{i=\infty} A_{x,i} t^i \tag{A8f}$$

$$A_y = \sum_{i=0}^{i=\infty} A_{y,i}t^i \tag{A8g}$$

$$A_z = \sum_{i=0}^{i=\infty} A_{z,i}t^i \tag{A8h}$$

The three position coordinates x,y,z along with the radius r, the parameter b, and the acceleration components A_x,A_y,A_z are assumed to be represented in some neighborhood of assumed values by Taylor series expansions. Taylor series or power series will be used interchangeably in this appendix which can be justified by Theorem 39, page 354 of reference 9. The sufficient conditions for convergence of the particular series are covered, at least for the two body case, by Schanzle (ref. 11) and should be easily extendable to the present case of a constant thrust and constant mass-flow-rate burn. In fact, the convergence of the series for the acceleration terms of equations (A3a) to (A3c) is trivial, being the multiplication of the series for the sine and cosine of $\dot{\theta}t$, which is absolutely convergent everywhere, by the series for $a(t)$, which is absolutely convergent for $t < m_o/\dot{m}$. (See ref. 9.) Of course, t cannot get as large as $m_o/\dot{m}$ which corresponds to the spacecraft mass becoming zero.

In order to obtain the solution to equations (A7a) to (A7c) it is necessary to evaluate the coefficients of equations (A8a) to (A8h). The three series for the acceleration terms (eqs. (A8f) to (A8h)) are determined from equations (A6) while the series for x, y, z, r, and b are determined by means of recursive equations. The following general formula will aid in this development (ref. 12),

$$\left(\sum_{i=0}^{i=\infty} p_i t^i\right)\left(\sum_{i=0}^{i=\infty} q_i t^i\right) = \sum_{i=0}^{i=\infty}\left(\sum_{k=0}^{k=i} p_k q_{i-k}\right)t^i \tag{A9}$$

where p and q are the coefficients of the general power series. The recursive equation for x can be developed as follows. Differentiating equation (A8a) twice gives

$$\ddot{x} = \sum_{i=0}^{i=\infty} i(i-1)x_i t^{i-2}$$

and then substituting into equation (A7a) along with equations (A8e) and (A8f) yields

$$\sum_{i=0}^{i=\infty} i(i-1)x_i t^{i-2} = -\mu\left(\sum_{i=0}^{i=\infty} b_i t^i\right)\left(\sum_{i=0}^{i=\infty} x_i t^i\right) + \sum_{i=0}^{i=\infty} A_{x,i}t^i$$

Changing indices on the first summation gives

$$\sum_{i=0}^{i=\infty} (i+1)(i+2)x_{i+2}t^i = -\mu\left(\sum_{i=0}^{i=\infty} b_i t^i\right)\left(\sum_{i=0}^{i=\infty} x_i t^i\right) + \sum_{i=0}^{i=\infty} A_{x,i}t^i \tag{A10}$$

or by use of equation (A9) it may be rewritten as

$$\sum_{i=0}^{i=\infty} (i+1)(i+2)x_{i+2}t^i = -\mu \sum_{i=0}^{i=\infty} \sum_{k=0}^{k=i} b_k x_{i-k} t^i + \sum_{i=0}^{i=\infty} A_{x,i}t^i \tag{A11}$$

Equating the coefficients of the nth power of t gives the recursive equation for the coefficients of the x series, that is

$$(n+1)(n+2)x_{n+2} = -\mu \sum_{k=0}^{k=n} b_k x_{n-k} + A_{x,n} \tag{A12}$$

The recursive equations for y and z are similar. Therefore, the three recursive equations for the coefficients of the x, y, and z series are

$$x_{n+2} = \frac{1}{(n+1)(n+2)}\left(-\mu \sum_{k=0}^{k=n} b_k x_{n-k} + A_{x,n}\right) \tag{A13a}$$

$$y_{n+2} = \frac{1}{(n+1)(n+2)}\left(-\mu \sum_{k=0}^{k=n} b_k y_{n-k} + A_{y,n}\right) \tag{A13b}$$

$$z_{n+2} = \frac{1}{(n+1)(n+2)}\left(-\mu \sum_{k=0}^{k=n} b_k z_{n-k} + A_{z,n}\right) \tag{A13c}$$

where x_0, y_0, z_0 are the components of its initial position and x_1, y_1, z_1, are the components of the initial velocity.

The recursive equation for r is found by differentiating equation (A2), that is

$$r\dot{r} = x\dot{x} + y\dot{y} + z\dot{z}$$

and by substituting equation (A8a) to (A8d) to yield

$$\left(\sum_{i=0}^{i=\infty} r_i t^i\right)\left(\sum_{i=0}^{i=\infty} (i+1) r_{i+1} t^i\right) = \left(\sum_{i=0}^{i=\infty} x_i t^i\right)\left[\sum_{i=0}^{i=\infty} (i+1) x_{i+1} t^i\right] + \left(\sum_{i=0}^{i=\infty} y_i t^i\right)\left[\sum_{i=0}^{i=\infty} (i+1) y_{i+1} t^i\right]$$

$$+ \left(\sum_{i=0}^{i=\infty} z_i t^i\right)\left[\sum_{i=0}^{i=\infty} (i+1) z_{i+1} t^i\right]$$

Multiplying the series together by equation (A9) and equating the nth power of t gives

$$\sum_{k=0}^{k=n} (k+1) r_{k+1} r_{n-k} = \sum_{k=0}^{k=n} (k+1)\left(x_{k+1} x_{n-k} + y_{k+1} y_{n-k} + z_{k+1} z_{n-k}\right)$$

Removing the nth term from the summations produces the recursive equation for the coefficient of the r series, that is

$$r_{n+1} = \frac{1}{(n+1) r_o}\left[(n+1)\left(x_{n+1} x_o + y_{n+1} y_o + z_{n+1} z_o\right)\right.$$
$$\left. + \sum_{k=0}^{k=n-1} (k+1)\left(x_{k+1} x_{n-k} + y_{k+1} y_{n-k} + z_{k+1} z_{n-k} - r_{k+1} r_{n-k}\right)\right] \qquad \text{(A14)}$$

where

$$r_o = \left(x_o^2 + y_o^2 + z_o^2\right)^{1/2}$$

$$r_1 = \frac{\left(x_o x_1 + y_o y_1 + z_o z_1\right)}{r_o}$$

The recursive equation for b is found by differentiating equation (A5), which gives

$$r\dot{b} = -3b\dot{r}$$

In a manner similar to that used to develop the recursive equation for r one obtains the recursive equation for b as

$$b_{n+1} = \frac{1}{(n+1) r_o}\left[-3(n+1) r_{n+1} b_o - \sum_{k=0}^{k=n-1} (k+1)\left(3 r_{k+1} b_{n-k} + b_{k+1} r_{n-k}\right)\right] \qquad \text{(A15)}$$

where

$$b_o = \frac{1}{r_o^3} \qquad \text{and} \qquad b_1 = \frac{-3b_o r_1}{r_o}$$

Through equations (A13), (A14), and (A15), the coefficients of the assumed series solution to the differential equations of motion can be found which lead directly to the state of the spacecraft at time t by evaluation of the power series. The only remaining development is the series expression for the three acceleration terms.

The power series expansions of the three acceleration functions are most easily accomplished by considering the product of two series, the first of which is $a(t)$. Expansion of the magnitude of the acceleration (eq. (A4)) yields

$$a(t) = \frac{\text{Thrust}}{m_o + \dot{m}t} = \text{Thrust} \sum_{i=0}^{i=\infty} (-1)^i \left(\frac{\dot{m}^i}{m_o{}^{i+1}}\right) t^i \tag{A16}$$

The second series are the direction cosines which can be rewritten by use of the general trigonometry summation formulas as

$$\left[\cos(\delta+\dot{\theta}t)\cos\alpha - \sin(\delta+\dot{\theta}t)\sin\alpha\cos\beta\right] = F_1\cos\dot{\theta}t + F_2\sin\dot{\theta}t$$

$$\left[\cos(\delta+\dot{\theta}t)\sin\alpha + \sin(\delta+\dot{\theta}t)\cos\alpha\cos\beta\right] = F_3\cos\dot{\theta}t + F_4\sin\dot{\theta}t$$

$$\left[\sin(\delta+\dot{\theta}t)\sin\beta\right] = F_5\cos\dot{\theta}t + F_6\sin\dot{\theta}t$$

where F_i is a constant and is given by

$$F_1 = \cos\alpha\cos\delta - \cos\beta\sin\alpha\sin\delta$$

$$F_2 = -\cos\alpha\sin\delta - \cos\beta\sin\alpha\cos\delta$$

$$F_3 = \sin\alpha\cos\delta + \cos\beta\cos\alpha\sin\delta$$

$$F_4 = -\sin\alpha\sin\delta + \cos\beta\cos\alpha\cos\delta$$

$$F_5 = \sin\beta\sin\delta$$

$$F_6 = \sin\beta\cos\delta$$

The definitions for $\sin\dot{\theta}t$ and $\cos\dot{\theta}t$ can be given in terms of a power series as

$$\sin \dot{\theta} t = \dot{\theta} t - \frac{\dot{\theta}^3 t^3}{3!} + \frac{\dot{\theta}^5 t^5}{5!} + \cdots = \sum_{i=0}^{i=\infty} \frac{\sin\left(\frac{i\pi}{2}\right) \dot{\theta}^i t^i}{i!}$$

$$\cos \dot{\theta} t = 1 - \frac{\dot{\theta}^2 t^2}{2!} + \frac{\dot{\theta}^4 t^4}{4!} + \cdots = \sum_{i=0}^{i=\infty} \frac{\cos\left(\frac{i\pi}{2}\right) \dot{\theta}^i t^i}{i!}$$

Therefore the three acceleration functions can be written as

$$A_x(t) = \left[\text{Thrust} \sum_{i=0}^{i=\infty} \frac{(-1)^i \dot{m}^i t^i}{m_o^{i+1}}\right] \left\{ \sum_{i=0}^{i=\infty} \left[F_1 \frac{\sin\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} + F_2 \frac{\cos\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} \right] t^i \right\}$$

$$A_y(t) = \left[\text{Thrust} \sum_{i=0}^{i=\infty} \frac{(-1)^i \dot{m}^i t^i}{m_o^{i+1}}\right] \left\{ \sum_{i=0}^{i=\infty} \left[F_3 \frac{\sin\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} + F_4 \frac{\cos\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} \right] t^i \right\}$$

$$A_z(t) = \left[\text{Thrust} \sum_{i=0}^{i=\infty} \frac{(-1)^i \dot{m}^i t^i}{m_o^{i+1}}\right] \left\{ \sum_{i=0}^{i=\infty} \left[F_5 \frac{\sin\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} + F_6 \frac{\cos\left(\frac{i\pi}{2}\right) \dot{\theta}^i}{i!} \right] t^i \right\}$$

Multiplication of the two series by the general formula of equation (A9) yields the desired power series expansion for the three acceleration functions as follows:

$$A_x(t) = \text{Thrust} \sum_{i=0}^{i=\infty} \left\{ \sum_{k=0}^{k=i} \left[\frac{F_1 \sin\left(\frac{k\pi}{2}\right) \dot{\theta}^k + F_2 \cos\left(\frac{k\pi}{2}\right) \dot{\theta}^k}{k!} \right] \left[\frac{(-1)^{i-k} \dot{m}^{i-k}}{m_o^{i-k+1}} \right] \right\} t^i$$

$$A_y(t) = \text{Thrust} \sum_{i=0}^{i=\infty} \left\{ \sum_{k=0}^{k=i} \left[\frac{F_3 \sin\left(\frac{k\pi}{2}\right) \dot{\theta}^k + F_4 \cos\left(\frac{k\pi}{2}\right) \dot{\theta}^k}{k!} \right] \left[\frac{(-1)^{i-k} \dot{m}^{i-k}}{m_o^{i-k+1}} \right] \right\} t^i$$

$$A_z(t) = \text{Thrust} \sum_{i=0}^{i=\infty} \left\{ \sum_{k=0}^{k=i} \left[\frac{F_5 \sin\left(\frac{k\pi}{2}\right) \dot{\theta}^k + F_6 \cos\left(\frac{k\pi}{2}\right) \dot{\theta}^k}{k!} \right] \left[\frac{(-1)^{i-k} \dot{m}^{i-k}}{m_o^{i-k+1}} \right] \right\} t^i$$

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TABLE I

DEFINITION OF INPUT PARAMETERS FOR PROGRAM VITAP

Program symbol	Mathematical symbol	Dimensions	Units	Definition
NOPT		12	None	Integer array denoting free control variables and selected target parameters, NOPT(1 to 6) corresponds to $\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o$; NOPT(K) = 0 for Kth control fixed and NOPT(K) = 1 for Kth control free where K = 1,2,. . .,6; NOPT(7 to 12) corresponds to $\psi_1,\psi_2,\ldots,\psi_6$; NOPT(K+6) = 0 for Kth constraint omitted; NOPT(K+6) = 1 for the Kth constraint considered
KOPT		6	None	Integer array denoting the specific target parameter chosen (see table II)
AIN		6	km, sec, or deg	Array of values for target parameters denoted by KOPT
CONI	$a_o,e_o,i_o,\omega_o,\Omega_o$	5	km or deg	Orbital elements of initial orbit; $a_o < 0$ for hyperbola
GS	$\vec{\alpha}_1$	6	deg or sec	Initial values (guesses) of controls $\alpha,\beta,\delta,\dot{\theta},t_b,\nu_o$ (see sketch 1); these values will vary or remain fixed depending on NOPT(1 to 6)
GL	$\vec{\lambda}_1$	6		Initial guesses on Lagrange multipliers; if not input, GL(1 to 6) = 1.
HP	$\Delta\alpha_i$	6	deg or sec	Increments of controls for numerical partial derivatives; if not input, HP(1 to 6) = 0.6, 0.6, 0.6, 0.006, 1, 0.6.
V1	$\Delta\alpha_i^*$	6	deg or sec	Maximum allowable step size for controls during Newton-Raphson iteration; if not input, V1(1 to 6) = 15., 15., 15., 0.0015, 50., 15.
NSTEPS		1	None	Integer denoting number of segments used for power series solution to equations of motion; if not input, NSTEPS = 10.
MASS	m_o	1	kg	Initial mass of spacecraft

TABLE I – Concluded

DEFINITION OF INPUT PARAMETERS FOR PROGRAM VITAP

Program symbol	Mathematical symbol	Dimensions	Units	Definition
DMASS	$\dot{m}$	1	kg/sec	Mass-flow rate
THR		1	kN	Thrust of the spacecraft propulsion system
PERJD		1	days	Julian date of periapsis passage on initial conic
REFJD		1	days	Reference Julian date for constraints of longitude, latitude, and true anomaly at a reference time; PERJD and REFJD need not be input if these constraints are not used
PER	θ	1	deg	Angular distance from periapsis to landing point (see sketch b)
SLAT		1	deg	Declination of subsolar point in areocentric equatorial coordinate system
SLON		1	deg	Right ascension of subsolar point; Input SLAT and SLONG only if KOPT(5) = 5
ERR	ϵ	1	None	Newton-Raphson convergence criteria (see eq. (12)); if not input, ERR = 10^{-6}
MAXIT		1	None	Integer denoting maximum number of iterations allowed; if not input, MAXIT = 50.
UMARS	μ	1	km^3/sec^2	Mars gravitational constant; if not input, UMARS = 42828.4
MODE		1	None	Program mode: 1 - normal forward targeting; 2 - forward targeting, inclination within bounds; 3 - backward targeting
BOUND		2	deg	Bounds on inclination for MODE = 2; BOUND (1) is lower bound

TABLE II

TARGET PARAMETER REQUEST KEYS

Input value	Input parameter					
	KOPT(1)	KOPT(2)	KOPT(3)	KOPT(4)	KOPT(5)	KOPT(6)
1	a	e	i	ω	Ω	f
2	$\frac{1}{a}$	r_a	[a]Latitude at reference date	[a]Longitude at reference date	[a]Longitude at reference date	[a]True anomaly at reference date
3	r_a	r_p	$\vec{B}\cdot\hat{T}$ (see sketch 3)	[a]Latitude at reference date	[a]Latitude at reference date	[a]Longitude at reference date
4	Orbital period	$\vec{B}\cdot\hat{R}$ (see sketch 3)		[b]Declination of $\vec{S}$	[b]Right ascension of $\vec{S}$	
5	V_∞			[c]Latitude of landing point, ϕ	[d]Lighting angle at landing point, G	

[a]Value of PERJD and REFJD must be input (see table I).

[b]$\vec{S} \equiv$ Incoming hyperbolic asymptote (see sketch 3).

[c]Value of PER must be input (see table I).

[d]Value of PER, SLAT, and SLON must be input (see table I).

TABLE III

SAMPLE OUTPUT FOR NORMAL TARGETING MODE

[Mode = 1]

```
$DAT
NOPT    =  1,  C,  1,  0,  1,  1,  1,  0,  1,  1,  C,  C,
KOPT    =  2,  1,  1,  1,  1,  1,
AIN     =  0.488878024932E-C4,  0.C,  0.35E+02,  0.13038E+03,  0.0,  0.0,
CONI    = -0.4939C96E+C4,  C.197C4E+01,  0.3531E+C2,  0.129069233511E+03,
           0.3C3C71822048E+C3,
GS      = -0.5E+C2,  C.9E+02,  0.2E+02,  0.0,  0.24E+04, -C.6E+02,
GL      =  0.1E+C1,  C.1E+01,  0.1E+C1,  0.1E+01,  0.1E+C1,  C.1E+01,
HP      =  0.6E+C0,  0.6E+0C,  0.6E+00,  C.6E-03,  0.1E+01,  0.6E+00,
V1      =  0.15E+02,  0.15E+C2,  C.15E+02,  C.15E-02,  C.5E+02,  0.15E+02,
NSTEPS  =  8,
MASS    =  0.320695E+C4,
DMASS   = -0.4772E+0C,
THR     =  0.13345E+C1,
PERJD   = I,
REFJD   =  0.0,
PER     = I,
SLAT    = I,
SLON    = I,
ERR     =  0.1E-C7,
MAXIT   =  50,
UMARS   =  0.428284E+05,
MODE    =  1,
BOUND   =  0.0,  0.18E+03,
$END
```

TABLE III – Continued

```
                                   VIKING TARGETING ANALYSIS PROGRAM (VITAP)

*****NORMAL TARGETING (MODE=1)*****

INITIAL CONIC
 SMA -4939.0960       ECC  1.9704000       INC  35.310000       PER  129.06923       NOD  303.07182       TAN -60.000000

THE CONTROL VARIABLES ARE      ALPHA       DELTA       TBURN       TASTART

THE TARGET VARIABLES ARE       1/SMA       INC         PERI



ITERATION    1

CONTROL PARAMETERS
         ALPHA                BETA                 DELTA                THDOT                TBURN                TASTART
     -50.0000000000       90.0000000000        20.0000000000        0.                   2400.0000000         -60.0000000000

LAGRANGE MULTIPLIERS
      1.00000000000     0.                     1.00000000000        1.00000000000       0.                   0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA  68636.103       ECC  .93090775       INC  33.327990       PER  139.78924       NOD  307.84094       TAN  37.890110

TARGET PARAMETERS
         1/SMA                ECC                  INC                  PERI                 NODE                 TAN
     1.456959179706E-05   .930907746419        33.3279901131        139.789236539        307.840943497        37.8901097288

DELTA V    1.2355143

ERRORS IN TARGET VARIABLES
         1/SMA                ECC                  INC                  PERI                 NODE                 TAN
     3.431821069614E-05   0.                   1.67200988694        -9.40923653906       0.                   0.

CORRECTIONS TO CONTROL VARIABLES
         ALPHA                BETA                 DELTA                THDOT                TBURN                TASTART
     -9.91246578435       0.                   6.10646498221        0.                   -17.6606928105       -15.0000000000

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
     7.9414928297D+01     2.0000437890D+00     -2.9815233394D+01     -5.5182930724D+01
```

TABLE III – Continued

ITERATION 2

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-59.9124657843	90.0000000000	26.1064649822	0.	2382.33930719	-75.0000000000

LAGRANGE MULTIPLIERS

-564455.121282	0.	8.40845983881	-.766108102041	0.	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA 132642.89 ECC .96780569 INC 36.227467 PER 138.67499 NOD 316.23571 TAN 351.64946

TARGET PARAMETERS

1/SMA	ECC	INC	PERI	NODE	TAN
7.539039663114E-06	.967805693215	36.2274669615	138.674993190	316.235705212	351.649461687

DELTA V 1.2241060

ERRORS IN TARGET VARIABLES

1/SMA	ECC	INC	PERI	NODE	TAN
4.134876283009E-05	0.	-1.22746696148	-8.29499319019	0.	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
2.65722330499	0.	6.777192974189E-02	0.	-50.0000000000	.142610856923

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

1.2652278826D+03	1.9535341856D+02	1.9999762086D+00	-1.1344834257D+02

ITERATION 29

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-25.6732438975	90.0000000000	22.5386393285	0.	2453.84069840	-63.8506518549

LAGRANGE MULTIPLIERS

-4783350555.7	0.	55950.9037335	101.077861597	0.	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA 20455.000 ECC .76436452 INC 35.000000 PER 130.38000 NOD 303.97940 TAN 40.321847

TARGET PARAMETERS

1/SMA	ECC	INC	PERI	NODE	TAN
4.888780249287E-05	.764364521465	35.0000000000	130.380000000	303.979400569	40.3218474563

DELTA V 1.2705839

ERRORS IN TARGET VARIABLES

1/SMA	ECC	INC	PERI	NODE	TAN
3.267785347871E-16	0.	-2.728484105319E-12	1.909938873723E-11	0.	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
3.012653474970E-11	0.	6.069949388672E-11	0.	3.215712718649E-09	-2.175990898215E-10

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

2.4341194253D+07	1.2883596478D+07	3.8334187041D+06	1.9648570252D+00

TABLE III – Concluded

NEWTON RAPHSCN MATRIX

1.267E+C7	0.	5.835E+C5	0.	-2.079E+C3	-7.553E+06	4.504E-06	0.	3.904E+00	-2.946E+01	0.	0.
0.	-2.878E-01	0.	0.	C.	0.	0.	0.	0.	0.	0.	0.
5.835E+05	0.	1.411E+07	0.	-1.9C5E+C3	-6.888E+C6	-8.788E-06	0.	-7.436E+0C	-4.256E+01	0.	0.
0.	0.	0.	-2.878E+05	C.	0.	0.	0.	0.	C.	0.	0.
-2.079E+03	0.	-1.905E+C3	C.	3.29CE+CC	4.330E+03	1.020E-07	0.	-4.834E-04	-5.321E-03	0.	0.
-7.553E+C6	0.	-6.888E+C6	C.	4.330E+03	1.428E+07	-1.486E-06	0.	-1.225E+00	-2.549E+01	0.	0.
4.504E-C6	0.	-8.788E-C6	0.	1.02CE-C7	-1.486E-C6	0.	0.	0.	C.	0.	0.
0.	0.	0.	0.	C.	C.	0.	0.	0.	C.	0.	0.
3.9C4E+C0	0.	-7.436E+C0	0.	-4.834E-04	-1.225E+00	0.	0.	0.	C.	0.	0.
-2.946E+C1	0.	-4.256E+01	0.	-5.321E-03	-2.549E+01	0.	0.	0.	C.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	C.	0.	0.
0.	0.	0.	0.	C.	0.	0.	0.	0.	C.	0.	0.

INVERSE

4.539D-C9	0.	4.479D-C9	0.	-1.93CD-16	-1.273D-08	4.429D+02	0.	1.044D-C1	-1.473D-02	0.	0.
0.	0.	0.	0.	0.	C.	0.	0.	0.	0.	0.	0.
4.479D-C9	0.	4.42CD-C9	0.	-1.9C5D-16	-1.256C-C8	2.538D+01	0.	-8.195D-C2	-5.622D-03	0.	0.
0.	0.	0.	0.	C.	C.	0.	0.	0.	0.	0.	0.
-1.930D-16	0.	-1.905D-16	0.	8.21CD-24	5.413C-16	9.747D+06	0.	-1.140C+01	-2.060D-02	0.	0.
-1.273D-C8	0.	-1.256C-C8	0.	5.413D-16	3.569C-08	-2.589D+03	0.	1.850C-C2	-1.281D-C2	0.	0.
4.429D+C2	0.	2.538D+C1	C.	9.747D+C6	-2.589D+03	-1.916D+14	0.	-3.277D+C8	9.418D+07	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	C.	0.	0.
1.044D-C1	0.	-8.195C-02	0.	-1.140D+C1	1.850C-02	-3.277D+08	0.	-2.195C+C5	1.019D+04	0.	0.
-1.473D-C2	0.	-5.622C-03	0.	-2.C6CC-02	-1.281D-02	9.418D+07	0.	1.C19D+C4	-1.792D+03	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	C.	0.	0.	0.	C.	0.	0.

TIME FOR THIS CASE 22.740

TABLE IV

SAMPLE OUTPUT FOR TARGETING WITHIN BOUNDS

[Mode = 2]

```
$DAT

NOPT    =  1,  1,  1,  1,  1,  1,  1,  1,  0,  1,  1,  C,

KOPT    =  1,  1,  1,  1,  1,  1,

AIN     =  0.20455E+C5,  C.776629372365E+0C,  0.C,  C.13C38E+03,  0.30327E+03,
           0.0,

CONI    = -0.4939096E+C4,  0.19704E+01,  C.3531E+C2,  0.129069233511E+03,
           0.303071822048E+03,

GS      = -0.55E+C2,  C.4E+C2,  C.5E+01,  0.25E-C1,  C.24E+C4, -0.6E+02,

GL      =  0.1E+C1,  0.1E+01,  0.1E+01,  C.1E+01,  0.1E+C1,  0.1E+C1,

HP      =  0.6E+CC,  C.6E+CC,  C.6E+00,  C.6E-C3,  0.1E+01,  C.6E+00,

V1      =  0.15E+02,  0.15E+02,  C.15E+02,  C.15E-02,  C.5E+02,  0.15E+C2,

NSTEPS  =  8,

MASS    =  0.32C695E+04,

DMASS   = -0.4772E+00,

THR     =  0.13345E+C1,

PERJD   = I,

REFJD   =  0.0,

PER     = I,

SLAT    = I,

SLON    = I,

ERR     =  0.1E-C7,

MAXIT   =  50,

UMARS   =  0.428284E+C5,

MODE    =  2,

BOUND   =  0.31E+C2,  C.35E+02,

$END
```

TABLE IV – Continued

```
                                   VIKING TARGETING ANALYSIS PROGRAM (VITAP)

*****INCLINATION BETWEEN BOUNDS (MODE=2)*****

INITIAL CONIC
 SMA -4939.0960      ECC  1.9704000      INC  35.310000      PER  129.06923      NOD  303.07182      TAN -60.000000

THE CONTROL VARIABLES ARE     ALPHA     BETA      DELTA     THDOT     TBURN     TASTART

THE TARGET VARIABLES ARE      SMA       ECC       PERI      NODE



ITERATION   1

CONTROL PARAMETERS
          ALPHA              BETA                DELTA                THDOT              TBURN              TASTART
     -55.0000000000     40.0000000000      5.0000000000      2.50000000000E-02   2400.0000000      -60.0000000000

LAGRANGE MULTIPLIERS
      1.0000000000      1.0000000000      0.                    1.00000000000      1.0000000000      0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA  19349.912      ECC  .76466673      INC  34.806452      PER  128.75868      NOD  303.52583      TAN  50.136035

TARGET PARAMETERS
          SMA                ECC                 INC                  PERI               NODE               TAN
      19349.9120075     .764666731799      34.8064515029       128.758679590      303.525826665      50.1360349024

DELTA V    1.2355143

ERRORS IN TARGET VARIABLES
          SMA                ECC                 INC                  PERI               NODE               TAN
      1105.08799253     1.196264056613E-02  0.                    1.62132041005     -.255826664554      0.

CORRECTIONS TO CONTROL VARIABLES
          ALPHA              BETA                DELTA                THDOT              TBURN              TASTART
     -11.8746860926     -15.0000000000      11.4010808779      -3.25938636540 8E-04  -15.2568751492     -2.43889172648

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
    2.31589477380+11    8.41282603010+04    5.62459016480+04    2.02037460520+04    1.76604104880+03    2.17771354990+00
```

TABLE IV – Continued

ITERATION 2

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-66.8746860926	25.0000000000	16.4010808779	2.467406136346E-02	2384.74312485	-62.4388917265

LAGRANGE MULTIPLIERS

3.11933539515	-10592.1550964	0.	42.1739821634	-9.18282948849	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA 20204.416 ECC .77402282 INC 36.239337 PER 131.02962 NOD 302.30496 TAN 43.941272

TARGET PARAMETERS

SMA	ECC	INC	PERI	NODE	TAN
20204.4162659	.774022817646	36.2393371786	131.029617172	302.304960945	43.9412715869

DELTA V 1.2256561

ERRORS IN TARGET VARIABLES

SMA	ECC	INC	PERI	NODE	TAN
250.583734108	2.606554719133E-03	0.	-.649617171922	.965039054750	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
15.0000000000	7.40529249771	-13.2390607551	-7.521978869204E-04	-10.2106029132	.669162760881

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

7.52956629850D+11	3.09311345920D+05	2.17434989280D+05	9.35847334110D+04	7.84860276920D+03	2.61596658610D+00

ITERATION 12

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-58.3595295367	34.9476030320	8.89468323481	2.407462446420E-02	2371.64763119	-61.7465949681

LAGRANGE MULTIPLIERS

94.2815423607	517787.364574	0.	182.143038384	-3404.67367318	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA 20455.000 ECC .77662937 INC 35.281270 PER 130.38000 NOD 303.27000 TAN 44.620900

TARGET PARAMETERS

SMA	ECC	INC	PERI	NODE	TAN
20455.0000000	.776629372365	35.2812702454	130.380000000	303.270000000	44.6208995264

DELTA V 1.2172220

ERRORS IN TARGET VARIABLES

SMA	ECC	INC	PERI	NODE	TAN
-4.656612873077E-10	-3.552713678801E-15	0.	-9.094947017729E-13	0.	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-7.381398630253E-09	-6.765011588708E-09	6.092481265466E-09	7.885268236581E-14	9.678599365652E-12	-1.637995933861E-10

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

2.50717982820D+13	9.75986655460D+06	6.96507590210D+06	2.56025473700D+06	2.49473610340D+05	2.23577477660D+01

***** INCLINATION NOT WITHIN BOUNDS - BEGIN TARGETING TO NEAREST BOUND*****

TABLE IV – Continued

```
$DAT

NOPT     =  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  1,  C,

KOPT     =  1,  1,  1,  1,  1,  1,

AIN      =  0.2C455E+C5,  0.776629372365E+0C,  0.35E+02,  0.13038E+03,
            0.30327E+C3,  0.0,

CONI     = -0.4939C96E+04,  0.19704E+01,  0.3531E+02,  0.129069233511E+03,
            0.3C3C71822C48E+C3,

GS       = -0.583595295440 41E+02,  0.349476030252C9E+02,  0.88946832409062E+01,
            0.24C746244 6428E-01,  0.23716476311911E+04, -C.61746594968311E+02,

GL       =  0.942815424CC895E+C2,  C.51778736129592E+C6,  C.C,
            0.182143038 51859E+C3, -0.34046736587503E+C4,  C.0,

HP       =  0.6E+00,  0.6E+00,  0.6E+00,  C.6E-03,  0.1E+01,  C.6E+C0,

V1       =  0.15E+C2,  C.15E+C2,  0.15E+02,  0.15E-02,  C.5E+02,  0.15E+C2,

NSTEPS   =  8,

MASS     =  0.32C695E+C4,

DMASS    = -0.4772E+00,

THR      =  0.13345E+C1,

PERJD    = I,

REFJD    =  0.0,

PER      = I,

SLAT     = I,

SLON     = I,

ERR      =  0.1E-C7,

MAXIT    =  50,

UMARS    =  0.428284E+05,

MODE     =  1,

BOUND    =  0.31E+C2,  C.35E+02,

$END
```

TABLE IV – Continued

```
                                     VIKING TARGETING ANALYSIS PROGRAM (VITAP)

*****NORMAL TARGETING (MODE=1)*****

INITIAL CONIC
 SMA -4939.0960       ECC  1.9704000      INC  35.310000       PER  129.06923       NOD  303.07182       TAN -61.746595

THE CONTROL VARIABLES ARE      ALPHA       BETA        DELTA       THDOT       TBURN       TASTART

THE TARGET VARIABLES ARE       SMA         ECC         INC         PERI        NODE



ITERATION   1

CONTROL PARAMETERS
          ALPHA                BETA                 DELTA                THDOT                TBURN                TASTART
     -58.3595295440       34.9476030252        8.89468324091       2.407462446428E-02   2371.64763119        -61.7465949683

LAGRANGE MULTIPLIERS
      94.2815424009       517787.361296        0.                        182.143038919        -3404.67365875        0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA  20455.000       ECC  .77662937      INC  35.281270       PER  130.38000       NOD  303.27000       TAN  44.620900

TARGET PARAMETERS
          SMA                  ECC                  INC                  PERI                 NODE                 TAN
      20455.0000000       .776629372365        35.2812702458        130.380000000        303.270000000        44.6208995261

DELTA V   1.2172220

ERRORS IN TARGET VARIABLES
          SMA                  ECC                  INC                  PERI                 NODE                 TAN
    3.259629011154E-09  3.907985046681E-14  -.281270245830     -2.728484105319E-12  1.818989403546E-12  0.

CORRECTIONS TO CONTROL VARIABLES
          ALPHA                BETA                 DELTA                THDOT                TBURN                TASTART
      4.53198691813        4.15337013317        -3.68688135516     -3.248484222292E-05 -1.088411549484E-09  8.109117220136E-03

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
    2.5071798290D+13    9.7598665634D+06    6.9650759127D+06    2.5602547486D+06    2.4947362108D+05    2.2357748672D+01
```

TABLE IV – Continued

```
ITERATION   2

CONTROL PARAMETERS
      ALPHA            BETA             DELTA              THDOT              TBURN            TASTART
   -53.8275426259   39.1009731584    5.20780188575    2.404213962206E-02  2371.64763119    -61.7384858511

LAGRANGE MULTIPLIERS
    93.8639059660    558372.117021    24012.2080778     138.015147139     -129.568496414    0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA  20482.257     ECC  .77689615     INC  35.035507     PER  130.54974     NOD  303.16178     TAN  44.570231

TARGET PARAMETERS
      SMA              ECC              INC                PERI               NODE               TAN
   20482.2571873    .776896150604    35.0355070207     130.549736793      303.161779428      44.5702307306

DELTA V   1.2172220

ERRORS IN TARGET VARIABLES
      SMA              ECC              INC                PERI               NODE               TAN
   -27.2571872891  -2.667782386716E-04 -3.550702073790E-02  -.169736792887  .108220572205     0.

CORRECTIONS TO CONTROL VARIABLES
      ALPHA            BETA             DELTA              THDOT              TBURN            TASTART
   -.435878270093   6.474164713686E-02  .363383497149   7.086660695794E-05  .692450922945    4.973723280717E-03

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
   2.5151378535D+13   9.6807204646D+06   7.2150754743D+06   2.3375250309D+06   3.6444220712D+05   2.2219739416D+01


ITERATION   6

CONTROL PARAMETERS
      ALPHA            BETA             DELTA              THDOT              TBURN            TASTART
   -54.2615624406   39.1678744022    5.56965351001    2.411239081703E-02  2372.37084545    -61.7341841275

LAGRANGE MULTIPLIERS
    93.8681130452    552067.961589    24375.509069      20.4146420811     -24.7008177262    0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA  20455.000     ECC  .77662937     INC  35.000000     PER  130.38000     NOD  303.27000     TAN  44.674292

TARGET PARAMETERS
      SMA              ECC              INC                PERI               NODE               TAN
   20455.0000000    .776629372365    35.0000000000     130.380000000      303.270000000      44.6742915574

DELTA V   1.2176872

ERRORS IN TARGET VARIABLES
      SMA              ECC              INC                PERI               NODE               TAN
   1.979060471058E-09  1.776356839400E-14  9.094947017729E-13 -9.094947017729E-13  0.            0.

CORRECTIONS TO CONTROL VARIABLES
      ALPHA            BETA             DELTA              THDOT              TBURN            TASTART
   -4.876928302904E-12 -1.005350177936E-11  2.465918164164E-11 -3.874074630498E-15 -4.588838924858E-11 -1.739523923412E-11

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
   2.5092868727D+13   9.6624449378D+06   7.2832766814D+06   2.3467554479D+06   3.2889508371D+05   2.2110569209D+01
```

TABLE IV – Concluded

NEWTON RAPHSON MATRIX

1.083E+07	-3.137E+06	1.014E+07	1.291E+10	6.567E+02	-7.090E+06	-6.753E+02	-1.437E-02	2.935E+00	-2.378E+01	-1.096E+01	0.
-3.137E+06	4.388E+06	-4.709E+04	-4.223E+07	-7.467E+02	-8.301E+04	1.636E+03	1.752E-02	-6.680E+00	-6.889E+00	1.066E+01	0.
1.014E+07	-4.709E+04	1.303E+07	1.661E+10	3.448E+02	-9.257E+06	1.933E+02	-6.076E-03	-5.701E-01	-4.167E+01	1.720E+00	0.
1.291E+10	-4.223E+07	1.661E+10	2.509E+13	2.416E+05	-1.194E+10	1.731E+05	1.927E+01	-1.058E+03	-5.240E+04	1.581E+03	0.
6.567E+02	-7.467E+02	3.448E+02	2.416E+05	2.305E+01	2.266E+03	-4.742E+01	-5.061E-04	-5.529E-04	-1.731E-02	8.341E-05	0.
-7.090E+06	-8.301E+04	-9.257E+06	-1.194E+10	2.266E+03	1.469E+07	1.286E+02	-1.022E-03	-4.468E-01	-3.115E+01	-1.016E+00	0.
-6.753E+02	1.636E+03	1.933E+02	1.731E+05	-4.742E+01	1.286E+02	0.	0.	0.	0.	0.	0.
-1.437E-02	1.752E-02	-6.076E-03	1.927E+01	-5.061E-04	-1.022E-03	0.	0.	0.	0.	0.	0.
2.935E+00	-6.680E+00	-5.701E-01	-1.058E+03	-5.529E-04	-4.468E-01	0.	0.	0.	0.	0.	0.
-2.378E+01	-6.889E+00	-4.167E+01	-5.240E+04	-1.731E-02	-3.115E+01	0.	0.	0.	0.	0.	0.
-1.096E+01	1.066E+01	1.720E+00	1.581E+03	8.341E-05	-1.016E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

INVERSE

2.681D-09	1.286D-09	2.924D-09	1.304D-12	-2.165D-17	-8.439D-09	6.669D-05	-6.139D+00	-2.293D-01	2.439D-03	-1.421D-01	0.
1.286D-09	6.172D-10	1.403D-09	6.258D-13	-1.039D-17	-4.049D-09	5.362D-05	-4.887D+00	-2.648D-01	3.600D-03	-6.989D-02	0.
2.924D-09	1.403D-09	3.188D-09	1.422D-12	-2.361D-17	-9.202D-09	4.408D-04	-4.120D+01	1.859D-01	-9.605D-03	1.106D-01	0.
1.304D-12	6.258D-13	1.422D-12	6.345D-16	-1.053D-20	-4.105D-12	-3.797D-07	3.575D-02	-6.651D-06	-5.285D-06	-7.972D-06	0.
-2.165D-17	-1.039D-17	-2.361D-17	-1.053D-20	1.749D-25	6.815D-17	-1.978D-02	-1.164D+02	-5.137D+00	-4.303D-03	5.206D-03	0.
-8.439D-09	-4.049D-09	-9.202D-09	-4.105D-12	6.815D-17	2.656D-08	-2.847D-06	8.171D-01	-1.051D-03	-1.302D-02	-1.065D-02	0.
6.669D-05	5.362D-05	4.408D-04	-3.797D-07	-1.978D-02	-2.847D-06	-5.778D-01	5.361D+04	3.849D+00	-7.589D+00	-5.352D+00	0.
-6.139D+00	-4.887D+00	-4.120D+01	3.575D-02	-1.164D+02	8.171D-01	5.361D+04	-5.061D+09	-3.234D+05	7.031D+05	5.052D+05	0.
-2.293D-01	-2.648D-01	1.859D-01	-6.651D-06	-5.137D+00	-1.051D-03	3.849D+00	-3.234D+05	-8.542D+04	9.398D+02	-1.105D+04	0.
2.439D-03	3.600D-03	-9.605D-03	-5.285D-06	-4.303D-03	-1.302D-02	-7.589D+00	7.031D+05	9.398D+02	-1.841D+03	-1.521D+03	0.
-1.421D-01	-6.989D-02	1.106D-01	-7.972D-06	5.206D-03	-1.065D-02	-5.352D+00	5.052D+05	-1.105D+04	-1.521D+03	-2.058D+04	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TIME FOR THIS CASE 29.532

TABLE V

SAMPLE OUTPUT FOR BACKWARD TARGETING MODE

[Mode = 3]

```
$DAT

NOPT     =  1,  1,  1,  1,  1,  1,  1,  0,  0,  1,  1,  0,

KOPT     =  5,  4,  3,  4,  4,  1,

AIN      =  0.29447E+01,  0.0,  0.0, -0.494E+01,  0.13008E+03,  0.0,

CONI     =  0.20455E+05,  0.776629372365E+00,  0.35E+02,  0.13038E+03,
            0.30327E+03,

GS       = -0.55E+02,  0.4E+02,  0.5E+01,  0.25E-01,  0.24E+04,  0.45E+02,

GL       =  0.1E+01,  0.1E+01,  0.1E+01,  0.1E+01,  0.1E+01,  0.1E+01,

HP       =  0.6E+00,  0.6E+00,  0.6E+00,  0.6E-03,  0.1E+01,  0.6E+00,

V1       =  0.15E+02,  0.15E+02,  0.15E+02,  0.15E-02,  0.5E+02,  0.15E+02,

NSTEPS   =  8,

MASS     =  0.320695E+04,

DMASS    = -0.4772E+00,

THR      =  0.13345E+01,

PERJD    = I,

REFJD    =  0.0,

PER      = I,

SLAT     = I,

SLON     = I,

ERR      =  0.1E-07,

MAXIT    =  50,

UMARS    =  0.428284E+05,

MODE     =  3,

BOUND    =  0.0,  0.18E+03,

$END
```

TABLE V – Continued

```
                                      VIKING TARGETING ANALYSIS PROGRAM (VITAP)

*****BACKWARD TARGETING (MODE=3)*****

INITIAL CONIC
 SMA  20455.000      ECC  .77662937      INC  35.000000      PER  130.38000      NOD  303.27000      TAN  45.000000

THE CONTROL VARIABLES ARE      ALPHA      BETA      DELTA      THDOT      TBURN      TASTART

THE TARGET VARIABLES ARE       VINF       DECSV     RTASV




ITERATION    1

CONTROL PARAMETERS
          ALPHA              BETA               DELTA              THDOT                TBURN              TASTART
     -55.0000000000     40.0000000000      5.0000000000       2.500000000000E-02   2400.0000000       45.0000000000

LAGRANGE MULTIPLIERS
      1.0000000000      0.                 0.                 1.0000000000         1.0000000000       0.

ORBIT OBTAINED FROM PRESENT CONTROLS
 SMA -4854.6847      ECC  1.9907407      INC  35.441364      PER  129.36505      NOD  302.82894      TAN  297.86829

TARGET PARAMETERS
          VINF               B*R                B*T                DECSV                RTASV              TAN
      2.97019807011     -4804.00183178     6837.73257053      354.674089349        130.355024490      297.868291354

DELTA V   1.2355143

ERRORS IN TARGET VARIABLES
          VINF               B*R                B*T                DECSV                RTASV              TAN
    -2.549807010624E-02  0.                 0.                 .385910651461        -.275024490129     0.

CORRECTIONS TO CONTROL VARIABLES
          ALPHA              BETA               DELTA              THDOT                TBURN              TASTART
      .482710002179      .457260657010     -2.03373774550      1.500000000000E-03   -4.40296290313     -.155157459117

 FINAL MASS       2061.670000

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION
    8.33241465410D+01    2.40839734570D+00    1.99999418160D+00    -3.72363234000D+00    -6.13476980160D+00    -6.27917766250D+06
```

TABLE V – Continued

ITERATION 2

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-54.5172899978	40.4572606570	6.66973188272	2.35000000000E-02	2395.59703710	45.1551574591

LAGRANGE MULTIPLIERS

2.30586197833	0.	0.	.979964684660	.990405855121	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA -4861.5682 ECC 1.9858033 INC 35.456375 PER 129.41727 NOD 302.83375 TAN 297.86383

TARGET PARAMETERS

VINF	B*R	B*T	DECSV	RTASV	TAN
2.96809456648	-4796.90561222	6823.25774489	354.689709502	130.333331173	297.863830483

DELTA V 1.2326658

ERRORS IN TARGET VARIABLES

VINF	B*R	B*T	DECSV	RTASV	TAN
-2.339456647724E-02	0.	0.	.370290498136	-.253331172693	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
.572259724437	.545786574855	-2.05866091386	1.500000000000E-03	-3.16638921221	-.116688788908

FINAL MASS 2063.771094

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

5.0960371867D+01	2.0000053276D+00	1.5233947392D+00	-4.7441646863D+00	-7.7915468208D+00	-1.0126775706D+07

ITERATION 46

CONTROL PARAMETERS

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
-58.9753864989	34.5884174529	16.2911605246	1.901992734839E-02	2367.96688861	44.5623489078

LAGRANGE MULTIPLIERS

-7460119.32293	0.	0.	9783.80393437	8895.53598726	0.

ORBIT OBTAINED FROM PRESENT CONTROLS

SMA -4939.1218 ECC 1.9596038 INC 35.050806 PER 129.30790 NOD 303.00276 TAN 297.70925

TARGET PARAMETERS

VINF	B*R	B*T	DECSV	RTASV	TAN
2.94469999956	-4743.85826518	6839.48432114	355.060000003	130.079999996	297.709246057

DELTA V 1.2148561

ERRORS IN TARGET VARIABLES

VINF	B*R	B*T	DECSV	RTASV	TAN
4.429097089087E-10	0.	0.	-3.044988261536E-09	4.349203663878E-09	0.

CORRECTIONS TO CONTROL VARIABLES

ALPHA	BETA	DELTA	THDOT	TBURN	TASTART
1.477348098775E-09	1.153097084863E-09	3.408362694156E-09	1.168156685974E-12	6.957999209935E-07	-9.931706836794E-09

FINAL MASS 2076.956201

EIGENVALUES OF SECOND PARTIALS OF AUGMENTED FUNCTION

1.9964323152D+13	1.0409077488D+07	5.8197725582D+06	1.9253265881D+06	2.3643011528D+05	3.7881902392D+00

TABLE V – Concluded

NEWTON RAPHSON MATRIX

9.642E+06	-3.036E+06	9.004E+06	9.753E+09	-5.326E+01	5.462E+06	1.340E-02	0.	0.	-5.643E+00	1.745E+01	0.
-3.036E+06	5.264E+06	-3.510E+04	-3.490E+07	-1.239E+01	1.989E+04	-1.111E-02	0.	0.	-5.290E+00	-3.496E+00	0.
9.004E+06	-3.510E+04	1.091E+07	1.183E+10	-7.768E+01	6.648E+06	5.376E-03	0.	0.	-1.201E+01	1.772E+01	0.
9.753E+09	-3.490E+07	1.183E+10	1.998E+13	-6.205E+05	6.970E+09	5.571E+00	0.	0.	-1.218E+04	1.806E+04	0.
-5.326E+01	-1.239E+01	-7.768E+01	-6.205E+05	3.918E+00	4.608E+02	6.352E-04	0.	0.	-1.480E-03	1.961E-03	0.
5.462E+06	1.989E+04	6.648E+06	6.970E+09	4.608E+02	6.760E+06	-2.079E-03	0.	0.	4.617E+00	-6.822E+00	0.
1.340E-02	-1.111E-02	5.376E-03	5.571E+00	6.352E-04	-2.079E-03	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-5.643E+00	-5.290E+00	-1.201E+01	-1.218E+04	-1.480E-03	4.617E+00	0.	0.	0.	0.	0.	0.
1.745E+01	-3.496E+00	1.772E+01	1.806E+04	1.961E-03	-6.822E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

INVERSE

1.268D-06	1.023D-06	-1.044D-06	-1.521D-12	-4.921D-15	2.070D-09	4.134D-02	0.	0.	2.052D-01	1.400D-01	0.
1.023D-06	8.262D-07	-8.441D-07	3.685D-14	-3.926D-15	1.555D-09	3.546D-03	0.	0.	3.515D-02	2.460D-02	0.
-1.044D-06	-8.441D-07	1.018D-06	-1.420D-10	-2.951D-16	3.193D-08	-3.125D-01	0.	0.	-1.885D-01	-8.681D-02	0.
-1.521D-12	3.685D-14	-1.420D-10	1.389D-13	5.417D-18	-4.958D-12	1.303D-04	0.	0.	1.653D-06	-1.529D-06	0.
-4.921D-15	-3.926D-15	-2.951D-16	5.417D-18	3.687D-22	3.006D-15	1.575D+03	0.	0.	-2.066D+00	-1.878D+00	0.
2.070D-09	1.555D-09	3.193D-08	-4.958D-12	3.006D-15	7.430D-08	9.020D-02	0.	0.	2.086D-02	-3.116D-02	0.
4.134D-02	3.546D-03	-3.125D-01	1.303D-04	1.575D+03	9.020D-02	-9.882D+06	0.	0.	-2.872D+04	4.107D+04	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2.052D-01	3.515D-02	-1.885D-01	1.653D-06	-2.066D+00	2.086D-02	-2.872D+04	0.	0.	-5.757D+04	-3.606D+04	0.
1.400D-01	2.460D-02	-8.681D-02	-1.529D-06	-1.878D+00	-3.116D-02	4.107D+04	0.	0.	-3.606D+04	-2.940D+04	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TIME FOR THIS CASE 77.756

NASA-Langley, 1971 — 30 L-7440

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